

Semiannual Progress Report, Jan.-June, 1980

(NASA-TM-81103) STUDY OF BOUNDARY-LAYER  
TRANSITION USING TRANSONIC-CONE PRESTON TUBE  
DATA Semiannual Progress Report, Jan. -  
Jun. 1980 (Oklahoma State Univ.,  
Stillwater.) 99 p HC A05/MF A01

N80-28305

CSCL 01A G3/02

Unclassified  
28139

STUDY OF BOUNDARY-LAYER TRANSITION

USING TRANSONIC-CONE PRESTON TUBE DATA

Research Grant Number NSG-2396

Principal Investigators

T. D. Reed and P. M. Moretti



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The NASA Technical Officer for this Grant is:

F. W. Steintle, Jr.

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## ACCOMPLISHMENTS

- I. An oral presentation was made on July 23 to Mr. F. W. Steinle and personnel of the Experimental Investigations Branch, NASA Ames. At that time, a review of technical progress was given.
- II. As discussed in detail within the accompanying report, a correlation of Preston-tube data with theoretical skin-friction coefficient has been achieved for the subsonic, compressible laminar boundary layers on the AEDC Cone. The recommended correlation has been developed using data from nineteen different wind-tunnel conditions and has an rms error in skin-friction coefficient of less than 5%.
- III. The STAN-5 computer program for boundary layer calculations is not sensitive to changes in cone pitch or yaw angles. Thus, if the effect of such angles on the correlation is to be studied, a more sophisticated analysis of three-dimensional, viscous flow will be needed, e.g., McRae, et al. [Ref. 1].
- IV. The simplified model for calculating the magnitude of Preston-tube pressures (as a function of boundary-layer profile, local static pressure, and probe geometry and position with respect to the wall) does not appear to be a fruitful approach. Thus a more rigorous analysis will be necessary if this type of sensitivity study is to be physically meaningful.
- V. An approach for developing a correlation for the subsonic, turbulent boundary layer and transitional region has been selected. Skin friction and velocity profiles, at the beginning of the turbulent boundary layer, can be estimated by using the correlation of Allen [Ref. 2] in conjunction with the Preston-tube data and the Wu and Lock and STAN-5 computer programs. Once the distribution of turbulent skin-friction and boundary layer profiles are available, a correlation between Preston-tube data and theoretical skin friction can be developed using the same techniques employed for the laminar boundary layer. Skin friction within the transition zone can be easily approximated by employing the empirical

intermittency function of Dharwan and Narasimha [Ref. 3]. Although this intermittency function is based on flat-plate measurements, the use of actual Preston-tube measurements to specify the extent of the transition zone will result in a very good approximation for the distribution of  $C_f$  through the transition zone.

VI. In the case of laminar boundary layers, there is no need to employ the more sophisticated program of Wilcox and Rubesin. However, this program may still be useful in checking the STAN-5 results for compressible, non-adiabatic turbulent boundary layers. This analysis and option will be relegated to future work.

VII. The supersonic wind-tunnel data cannot be successfully analyzed without a calibration of  $P_{ref}$  as a function of Preston-tube position,  $M_\infty$  and  $Re_{ft}$ . The corresponding calibrations for the flight experiments could conceivably be utilized, but this analysis will also be relegated to future work.

REMAINING TASKS TO BE ACCOMPLISHED  
UNDER THIS GRANT

- I. The effects of changes in nose bluntness on pressure distribution along the AEDC cone will be investigated.
- II. Subsonic Preston-tube data will be used to study and compare the onset and extent of boundary layer transition for the corresponding flight and wind-tunnel flow conditions.
- III. Use the flight data to develop a correlation for subsonic laminar boundary layers, with and without heat transfer, and compare the results with the corresponding correlation of the wind-tunnel data. The pressure distribution, measured during flight, will be used to calculate the flow, rather than the theoretical pressures of Wu and Lock.

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2. Allen, J. M.: "Reevaluation of Compressible Flow Preston-Tube Calibrations," NASA TM X-3488, February, 1977.
3. Dharwan, S. and Narasimha, R.: "Some Properties of Boundary Layer Flow During Transition from Laminar to Turbulent Motion," JFM, Vol. 3, Pt. 4, 1958, pp. 418-436.
4. Higuchi, H. and Rubesin, M. W.: "Behavior of a Turbulent Boundary Layer Subjected to Sudden Transverse Strain," AIAA Journal, Vol. 17, No. 9, Sept. 1979.

CORRELATION OF THEORETICAL LAMINAR SKIN  
FRICTION WITH PRESTON-TUBE  
MEASUREMENTS ON A  
SUBSONIC CONE

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MASTER OF SCIENCE  
May 1980

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## ABSTRACT

The laminar boundary layer on a 10-degree cone in a transonic wind tunnel is studied. The inviscid flow and boundary layer development are simulated by computer programs. The effects of pitch and yaw angles on the boundary layer are examined.

Preston-tube data, taken on the Arnold Engineering Development Center (AEDC) Boundary-Layer-Transition Cone in the NASA Ames 11-ft Transonic Wind Tunnel, has been used to develop a correlation which relates the measurements to theoretical values of laminar skin friction. The recommended correlation is based on a compressible form of the classical law-of-the-wall.

The computer codes successfully simulate the laminar boundary layer for near-zero pitch and yaw angles. However, in cases of significant pitch and/or yaw angles, the flow is three-dimensional and the boundary layer computer code used here cannot provide a satisfactory model.

The skin-friction correlation is thought to be valid for body geometries other than cones. It accounts for variable property and heat transfer effects. The rms deviation between theoretical skin-friction coefficients and the corresponding correlation values is < 5 %. Thus, as perhaps

might be expected, this is a better correlation for compressible laminar flows than has been reported for compressible, turbulent layers. The new correlation can be employed in transonic-wind-tunnel tests to relate Preston-tube surveys along models to distributions of laminar, skin-friction coefficient.

#### ACKNOWLEDGMENTS

I am most grateful to Dr. Peter M. Moretti, my principal adviser, for giving me the opportunity to work on this project and for his helpful suggestions and advice during the course of this study.

I am greatly indebted to Dr. Troy D. Reed for his continuous supervision and excellent guidance.

I would like also to thank Dr. Lynn R. Ebbesen and the staff of the University Computer Center for their help in the development of the computer programs.

This study was arranged through a NASA Ames University Consortium Interchange NCA2-OR535-701, the financial support of which is greatly acknowledged.

REPORT OF RESEARCH  
IN AERONAUTICS  
NO. 100-1000-1

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## NOMENCLATURE

$c_f$	Local skin-friction coefficient	
$c_p$	Specific heat at constant pressure, = 0.24 Btu/lbm°R for air	
$C_p$	Preston-tube pressure coefficient, = $(P_{pt} - P_w) / (0.5 \rho_e U_e^2)$	
$D$	Characteristic dimension of the probe, in.	
$D_{eq}$	Equivalent circular diameter of the probe, in.	
$f'$	Blasius velocity ratio, = $u/U_e$	
$G$	Gain factor for the pressure transducer, psi/in.	
$g_c$	Conversion factor, = 32.174 lbm·ft/lbf·s	
$h$	Enthalpy, Btu/lbm	
$H$	Pressure head, in.	
$J$	Mechanical to thermal energy conversion factor, = 778.2 lbf·ft/Btu	
$k$	Non-dimensional normal distance, = $2y/D$	
$L$	Cone axial length, in.	
$M$	Mach number	
$P$	Pressure, psi	ORIGINAL PAGE IS OF POOR QUALITY
$Pr$	Prandtl number	
$q$	Dynamic pressure, psi	
$r$	Recovery factor, or radial distance, in.	
$R$	Gas constant, = 53.35 lbf·ft/lbm°R for air	
$R_D$	Reynolds number based on D and $U_w$ , = $U_e D / U_w$	
$Re_{ft}$	Freestream unit Reynolds number, = $U_\infty / v_\infty$	

$Re_x$	Length Reynolds number, $= U_e x / \nu_e$
$Re_\theta$	Momentum-thickness Reynolds number, $= U_e \theta / \nu_w$
T	Temperature, $^{\circ}$ R
$T'$	Reference temperature, $^{\circ}$ R
u	Longitudinal velocity inside boundary layer, ft/s
$u_{Pt}$	Mean velocity across probe face, ft/s
$u^*$	Shear velocity, $= \sqrt{\tau_w / \rho_w}$
$u^+$	Normalized velocity for wall-law, $= u/u^*$
U	Velocity outside boundary layer, ft/s
x	Distance along cone surface, ft
X	Body force per unit volume, lbf/ft
$x^*$	Dimensionless independent variable, Eqn. (8.13)
y	Distance normal to cone surface, ft
$y^*$	Dimensionless dependent variable, Eqn. (8.8b)
$y^+$	Wall Reynolds number, $= y u^* / \nu_w$

#### Subscripts

aw	adiabatic wall
B	Blasius solution
c	for cone flow
e	at edge of boundary layer
eff	effective
eq	equivalent
E	external or outer
I	internal or inner
Pt	Preston-tube
ref	reference
s	shorted

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t total  
w at wall or cone surface  
W for wedge flow  
o wind-off  
∞ freestream condition

**Greek Letters**

$\alpha$  Angle of attack, deg.  
 $\bar{\alpha}$  Effective angle of attack, deg.  
 $\beta$  Yaw angle, deg., or pressure gradient parameter  
 $\gamma$  Ratio of specific heats, = 1.4 for air  
 $\delta$  Cone semi-vertex angle, deg., or boundary layer thickness, ft  
 $\delta^*$  Displacement thickness of boundary layer, ft  
 $\Delta$  Deflection or increment  
 $\epsilon$  Azimuth angle, deg.  
 $\eta$  Blasius non-dimensional normal distance, =  $y \sqrt{U_e / 2 \times \nu'}$   
 $\theta$  Momentum thickness of boundary layer, ft  
 $\mu$  Molecular viscosity, lbf-s/ft  
 $\nu$  Kinematic viscosity,  $\text{ft}^2/\text{s}$   
 $\nu'$  Kinematic viscosity evaluated at the reference temperature,  $\text{ft}^2/\text{s}$   
 $\rho$  Density,  $\text{lbm}/\text{ft}^3$   
 $\tau$  Shear stress, psf  
 $\phi$  Angle between cone axis and resolved yaw vector, deg.  
 $\psi$  Stream function,  $\text{lbm/s}$   
 $\omega$  Normalized stream function, Eqn.(6.1)

## CHAPTER I

### INTRODUCTION

The overall objective of this research is a better understanding of boundary layer transition, as reflected in the capability to relate transition on models in transonic wind tunnels to the corresponding free-flight conditions. The particular objective of the work reported herein is to develop a correlation which relates Preston-tube measurements within a laminar boundary layer on a cone to the corresponding theoretical values of skin friction.

Preston-tube measurements along the surface of a sharp 1.0-degree cone were obtained in the NASA Ames 11-Ft Transonic Wind Tunnel [1]. The minimum and maximum pressure locations, obtained during a survey along the length of the cone, were interpreted as the onset and end of transition, respectively.

The boundary layer on the slender cone was simulated via the STAN5 computer code [2] which is an extended version of Patankar and Spalding's boundary layer program [3]. The inviscid flow was calculated with Wu and Lock's computer program [4], and the results were used as boundary conditions along the outer edge of the boundary layer. Subroutines were added to this program so that arbitrary combina-

tions of pitch and yaw angles can be input, and the pressure distribution along the ray corresponding to the Preston-tube survey is always generated. In addition, a subroutine was added to the Wu and Lock program to calculate the initial profiles needed for STANS.

The cone is assumed to be stationary, smooth and sharp-nosed. The probe is assumed to be stable, in contact with the cone surface, and lie totally inside the boundary layer. The flow is assumed to be axi-symmetric, adiabatic, compressible and without body forces. The flow outside the boundary layer is assumed to be inviscid and is calculated based on the cone geometry, i.e., viscous interaction is ignored. The study was restricted to laminar boundary layers on the cone at subsonic speeds.

The effect on the inviscid flow of yaw and pitch angles less than the cone semi-vertex angle is easily calculated with the Wu and Lock program. However, the STANS program is a two-dimensional boundary layer code and was found to be relatively insensitive to changes in these angles.

A least-squares curve-fitting program [10] was used to arrive at a simple correlation between skin friction and Preston-tube measurements for the laminar, subsonic boundary layer.

## CHAPTER II

### OBJECTIVES

The first objective of this study was to calculate the best possible initial profiles, which are required to begin numerical boundary-layer calculations, so that boundary-layer predictions would be uniformly accurate. In an earlier study by Huprikar [5], it was found that different starting profiles resulted in differences in the computed shear stress near the tip of the cone. An example of this is shown in Figure 1.

The second objective was to extend the functions of Wu and Lock's program [4], which calculates the inviscid pressure distribution on sharp cones at transonic Mach numbers, so as to automate calculation of the pressure distribution along a ray corresponding to the Preston-tube survey for non-zero pitch ( $\alpha$ ) and yaw ( $\beta$ ) angles. This information then provides the inviscid boundary conditions for calculation of the boundary layer with STANS. The third objective was to obtain a correlation for skin-friction coefficient or wall-shear stress in terms of the Preston-tube pressure measurements, so that the Preston tube can be used as a skin-friction measuring device.

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The present research focuses on the NASA Ames wind tunnel data taken within laminar boundary layers on the AEDC Transition Cone at subsonic speeds.

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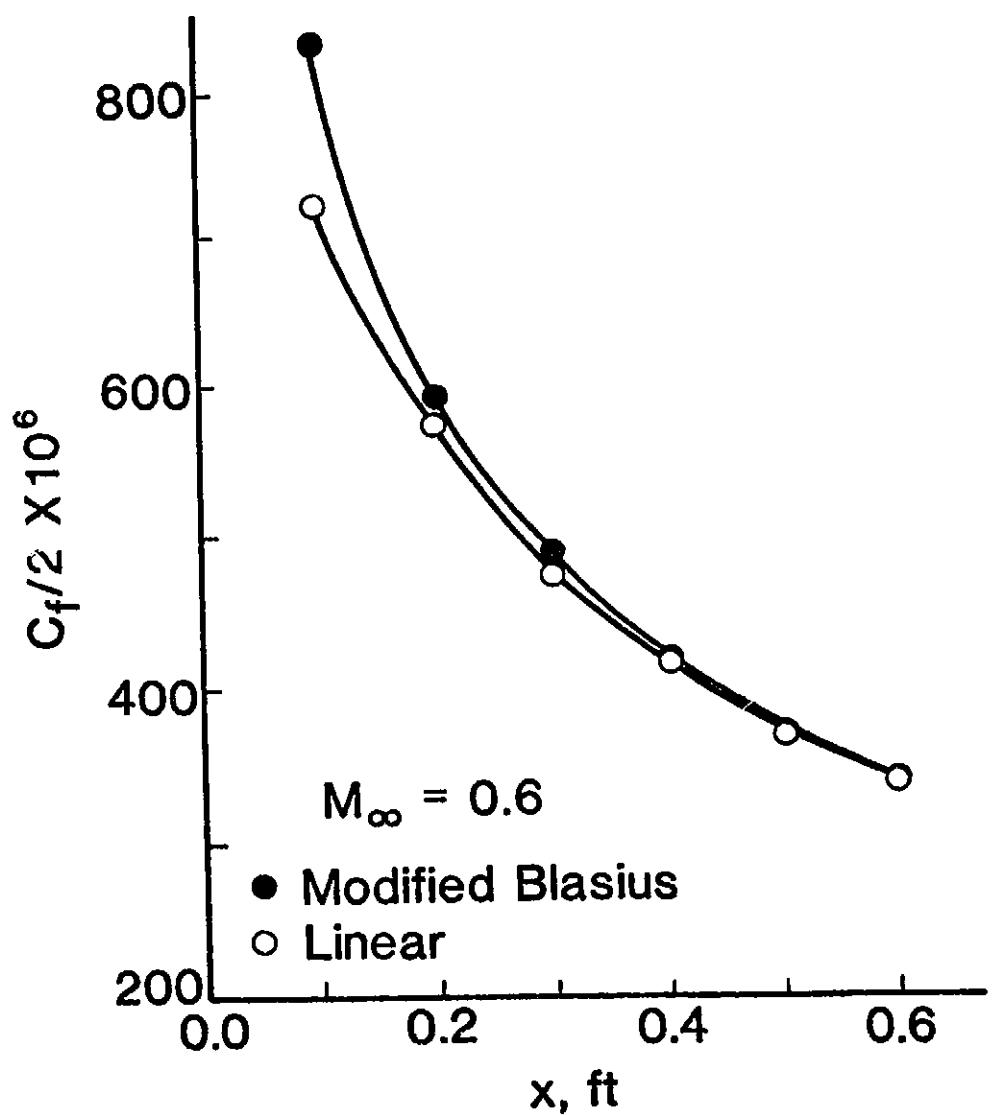


Figure 1. Effect of Initial Profile on Laminar Shear Stress Computation

## CHAPTER III

### EXPERIMENTAL DATA

#### 3.1 General Background

The measurements utilized in this research were obtained in the NASA 11-Ft Transonic Wind Tunnel at Moffet Field, California. A transonic wind tunnel is an experimental facility intended to simulate the flow over scaled, aerodynamic-test models that would be similar to full-scale vehicles during free-flight through the atmosphere at Mach numbers from approximately 0.5 to 1.5.

In transonic flow the difference between the freestream velocity and the speed of sound is small compared to the magnitude of either, and the changes in these parameters are of comparable magnitude. This is contrasted to subsonic flow, where the velocity is lower than the sonic speed and where changes in Mach number are primarily due to changes in freestream velocity at essentially constant sonic speeds, and to supersonic flow where the magnitude of the freestream velocity is substantially larger than the local sonic speed with changes in Mach number occurring through variations of both parameters. In the transonic Mach number range, not only do compressibility effects become important, compared

to lower subsonic Mach numbers where the flow is incompressible, but also the flow at near-sonic speeds is complex because of the mixed type of flow which may exist with local supersonic flow fields contained in subsonic flow regions or local subsonic flow fields embedded in supersonic flow regions. That is why the cone shape is used as a model for boundary-layer-transition research; since it will not have local shocks along the conical surface. At high subsonic speeds, a shock may be generated near the base of the cone owing to flow expansion at the rear of the conical surface and a subsequent recompression in the wake. At supersonic speeds, the cone will, of course, also generate a bow shock, but a shock does not occur on the surface throughout the subsonic Mach number range.

It is worth mentioning that the ventilated, test-section walls of a transonic wind tunnel introduce acoustic and streamline disturbances into the test-section flow which means that the wind tunnel flow does not correspond exactly to transonic, free-flight conditions [6]. No satisfactory method has yet been derived to correct for all of the wall effects [7], although this is an area of active research [8].

### 3.2 Apparatus and Measurements

The experimental data were obtained from a Pitot probe that was traversed longitudinally along the surface of a 5-degree half-angle cone. The cross-section of the opening of the

probe is shown in Figure 2 193. The opening has an oval shape with the small dimension normal to the cone surface. The outer height of the probe face is 0.0097", while the centerline of the opening is 0.00453" above the cone surface. A schematic of the experimental model and instrumentation is shown in Figure 3.

The total pressure, as sensed by the Pitot probe, was measured by a differential pressure transducer. The reference pressure for the transducer was taken from the static holes on a flow-angularity probe mounted underneath the cone.

The output from the pressure transducer,  $\Delta H$ , was recorded, during constant wind tunnel conditions, as a function of  $x$  on a plotter. Shorted output of the transducer, for the same wind tunnel conditions, was also plotted on the same plot. The output of the transducer, when the tunnel was off and the transducer was shorted, was also plotted. This output should theoretically be zero. This deflection is called 'wind-off' deflection.

Using this information, the total pressure  $P_{pt}$ , as measured by the Pitot probe can be deduced by using the relation

$$P_{pt} = P_{ref} + G(\Delta H + \Delta H_s + \Delta H_0) \quad (3.1)$$

Here  $P_{ref}$  is the reference static pressure which is considered to be equal to the freestream static pressure [9].  $\Delta H$

is the deflection of the plotter corresponding to the magnitude of  $\Delta P$  as sensed by the differential pressure transducer. The deflection from the shorted output is  $\Delta H_S$ , and  $\Delta H_0$  is the wind-off deflection.  $G$  is the gain factor of the plotter, and its value is 0.2515 psi/in. This value was determined from the calibration of the plotter [5].

Twenty-one cases were chosen for detailed analysis. These are all the available, subsonic-wind-tunnel cases with near-zero flow angles. The tabulated data for these runs is shown in Table I. The freestream Mach number, unit Reynolds number and dynamic pressure are given by  $M_\infty$ ,  $Re_{ft}$  and  $q_\infty$ , respectively, while  $\alpha$  and  $\beta$  are the angles of attack and yaw, respectively.

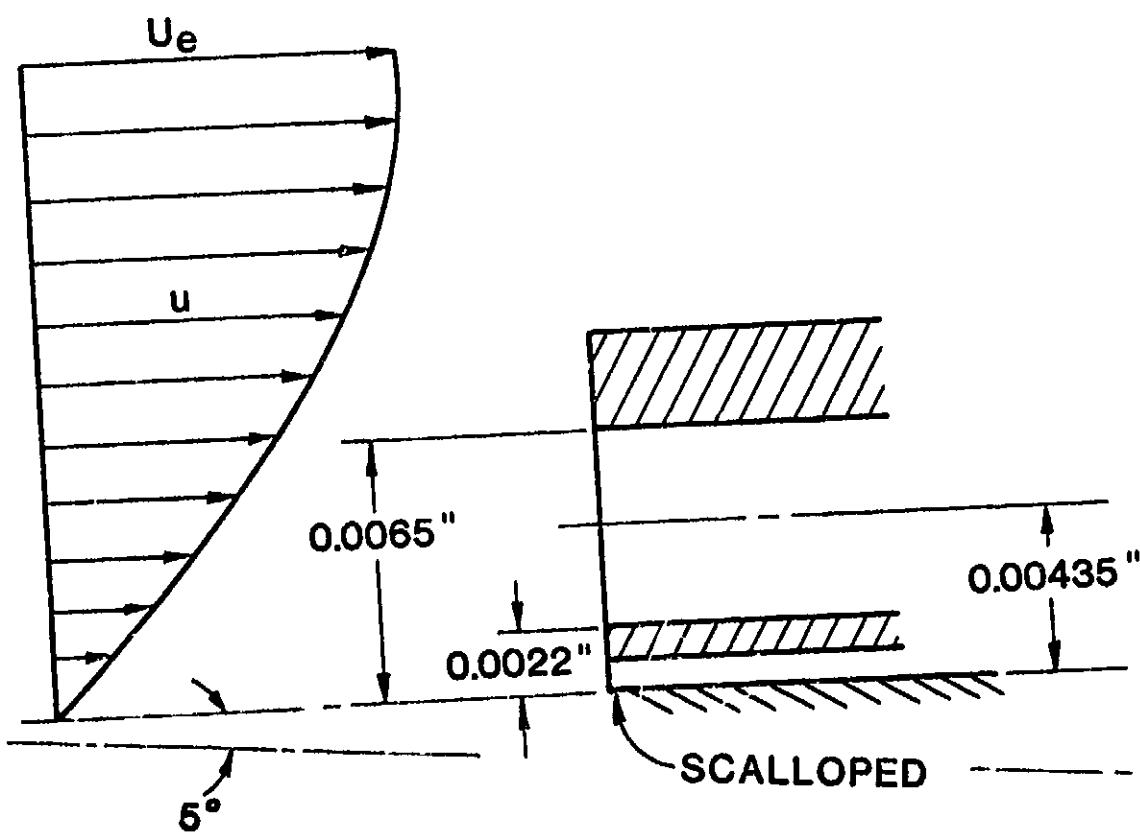
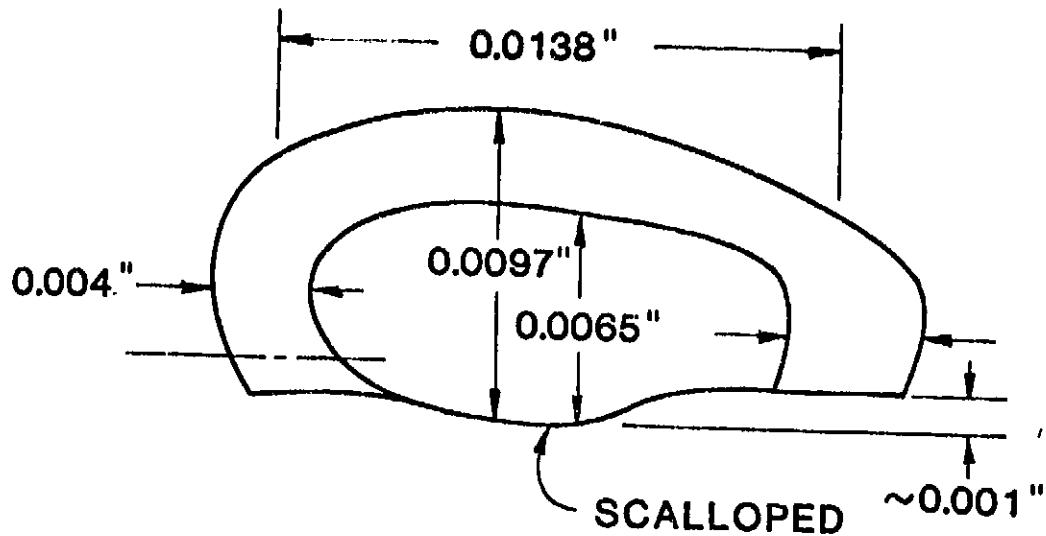


Figure 2. Geometry of the Probe

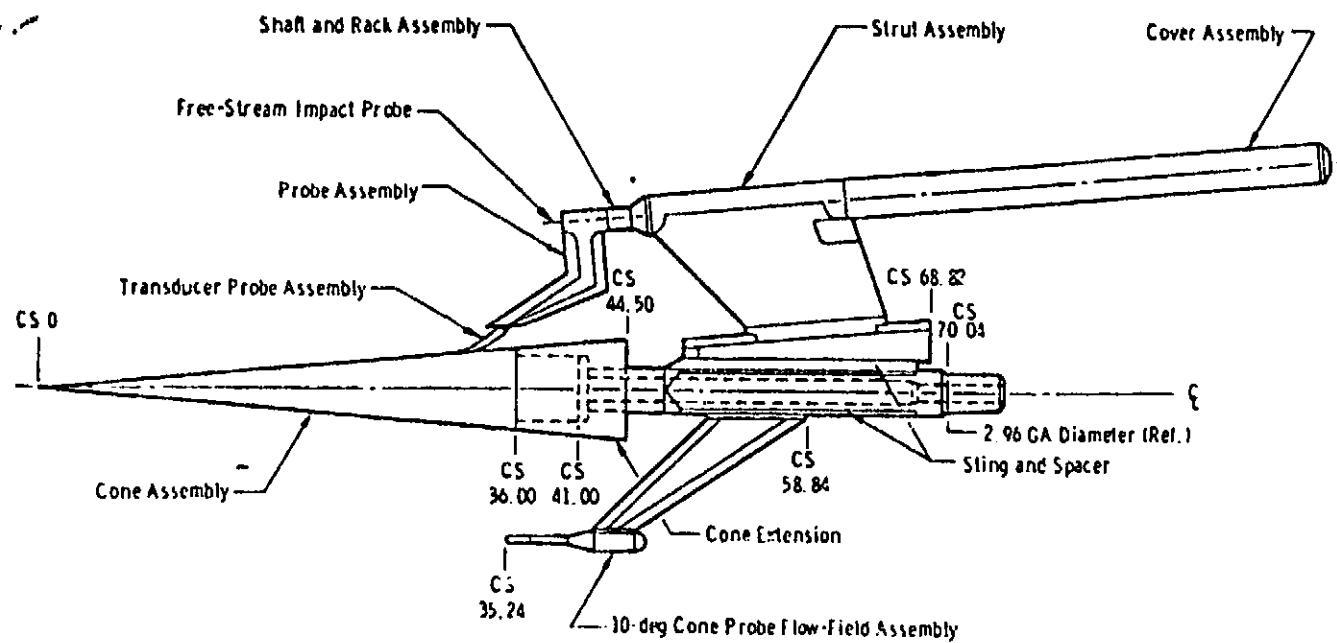


Figure 3. AEDC Transition Cone and Instrumentation

TABLE I  
CASES STUDIED

<u>RUN NO.</u>	<u><math>M_\infty</math></u>	<u><math>Re_{ft} \times 10^{-6}</math></u>	<u><math>q_\infty</math></u>	<u><math>\alpha^\circ</math></u>	<u><math>\beta^\circ</math></u>
15.231	0.95	4	693	-0.048	0.018
19.289	0.8	4	617	-0.003	-0.022
21.318	0.7	4	548	-0.006	-0.025
23.346	0.6	4	477	-0.001	-0.025
25.376	0.5	4	404	-0.005	-0.025
27.411	0.4	4	403	-0.004	-0.026
29.440	0.3	4	230	-0.006	-0.026
39.545	0.4	2.5	396	0.023	0.021
40.547	0.6	5	586	0.021	0.021
41.548	0.7	5	680	0.018	0.021
42.549	0.8	5	761	0.013	0.021
43.550	0.9	5	842	0.010	0.021
44.551	0.95	5	873	0.008	0.021
56.631	0.9	3	492	0.062	0.006
57.632	0.8	3	453	0.066	0.006
58.633	0.7	3	408	0.071	0.006
59.634	0.6	3	357	0.075	0.006
60.635	0.5	3	302	0.068	0.007
61.636	0.4	3	246	0.070	0.007
70.726	0.7	4	538	0.036	0.023
72.748	0.8	4	605	0.030	0.023

## CHAPTER IV

### CALCULATION PROCEDURE

The calculation procedure consists of the following steps for each case studied :

1. The given freestream parameters ( $M_\infty$ ,  $Re_{ft}$ ,  $q_\infty$ ), as well as the flow angles ( $\alpha$ ,  $\beta$ ), are fed into the extended Wu and Lock program. This program is described in the next chapter and is listed in Appendix F. The output is two-fold:

- a. The inviscid velocity distribution along the cone, and
- b. The initial profiles of velocity and stagnation enthalpy at a distance very close to the tip of the cone.

2. These results are then input to the STANS program. A brief description of STANS is presented in Chapter VI. The output from that program has detailed information on the boundary-layer properties along the ray of the cone which corresponds to the Preston-tube survey.

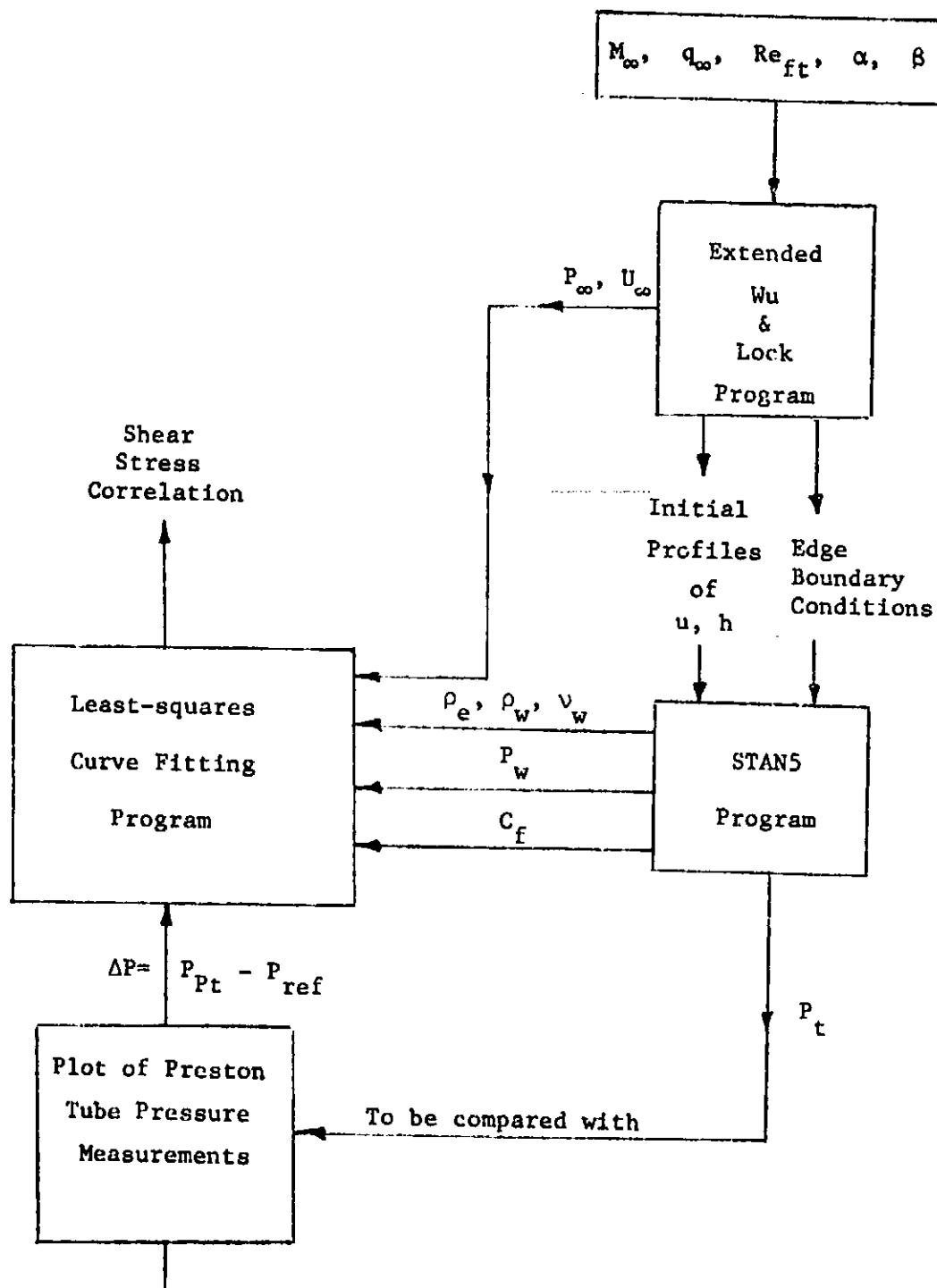
When correlation of skin friction was pursued, two more steps were followed :

3. Experimental Preston-tube pressure measurements were calculated from NASA/Ames 11 THT Preston-tube data [13] using Equation (3.1).

4. These experimental pressures, together with some other parameters\* calculated by STANS, were fed into a curve-fitting program [10] to obtain the required correlation. Figure 4 is a flow chart that summarizes the calculation procedure described above.

-----

\*See Chapter VIII for details.



1. Effective center of probe
2. Effect of  $\alpha, \beta$
- ... etc.

Figure 4. Flow Chart for the Analysis

## CHAPTER V

### EXTENDED WU AND LOCK COMPUTER PROGRAM

#### 5.1 Introduction

Wu and Lock [4] developed a computer program to calculate the inviscid transonic flow field over a sharp-edge smooth cone surface. The program appears to give accurate results [9] when compared with experimental observations. The program, however, handles only yaw angles less than the cone semi-vertex angle. The modified program presented herein calculates the following additional information :

1. The inviscid velocity and pressure distribution along a ray of the cone that corresponds to the Preston-tube survey for arbitrary combinations of  $\alpha$  and  $\beta$ ,
2. The effect of yaw angles on the inviscid flow field, and
3. The velocity and total enthalpy profiles at a user-specified initial station.

The listing of the extended program can be found in Appendix F. Output for an example run (Case 25.376) is also included.

#### 5.2 The Original Program

The main program reads in  $M_\infty$ ,  $\alpha$ ,  $\beta$ , and the cone semi-vertex angle  $\delta$  and calculates the inviscid-flow pressure

distribution along the cone surface. This is used as the pressure at the edge of the boundary layer. The theory and equations used are described in the Wu and Lock report [4]. The program prints out, along the cone length, the local Mach number  $M_e$  and the ratio  $P_w/P_\infty$ .

### 5.3 Subroutine ANGLES

This subprogram uses the angle-of-attack  $\alpha$  and the yaw angle  $\beta$  to calculate the effective pitch angle  $\bar{\alpha}$  and the azimuthal position of the probe  $\epsilon$ . This subroutine utilizes the equations derived by Dunn et al [11]. These equations are presented in Appendix A. The probe position is considered to be always at the top of the cone and, in accordance with Wu and Lock's notation,  $\epsilon = 0.0$  always corresponds to the leeward side of the cone. The calculated angles ( $\bar{\alpha}$  and  $\epsilon$ ) are then used in the main program to calculate the inviscid pressure distribution along the top of the cone.

### 5.4 Subroutine DIST

This subroutine reads in the freestream dynamic pressure (QTINF), unit Reynolds number (REFT) and Mach number (MINF). It then uses these values to calculate the freestream properties (PINF, TINF, RHOINF, MUSINF) as well as the total temperature and pressure (TTOT, PTOT). The equations used are the equation of state for a perfect gas (air), Sutherland's equation of viscosity and the isentropic relations [12]. The

details of the calculations are described in Appendix B.

Next, the subroutine uses the local Mach numbers at stations along the cone surface, which are calculated in the main program, to calculate the local temperatures and velocities using isentropic relations. These velocities are then used as the outer boundary conditions for calculation of the boundary layer using STAN5.

### 5.5 Subroutine INITIA

This last subprogram calculates the velocity and stagnation enthalpy profiles across the boundary layer at a specified initial location. It calculates the average static temperature and viscosity across the boundary layer and uses them to modify the flat-plate Blasius solution so as to apply to the cone problem. The details are presented in Appendix C.

### 5.6 Checking Wu and Lock Calculations

As a check on the reliability of our version of Wu and Lock's program, the inviscid flow was calculated for a 10-degree cone at a 2-degree pitch angle and compared with those in Wu and Lock's report [4]. The following observations were made :

- a. Static pressures on the windward side of the cone are larger than those on the leeward side.
- b. Increasing  $\alpha$  increases the static pressure on the windward side and decreases it on the leeward side.
- c. The slope of the pressure distribution is essentially

the same on both the windward and leeward sides of the cone.  
(except near the tip and the rear ends of the cone). These  
checks are shown in Figure 5.

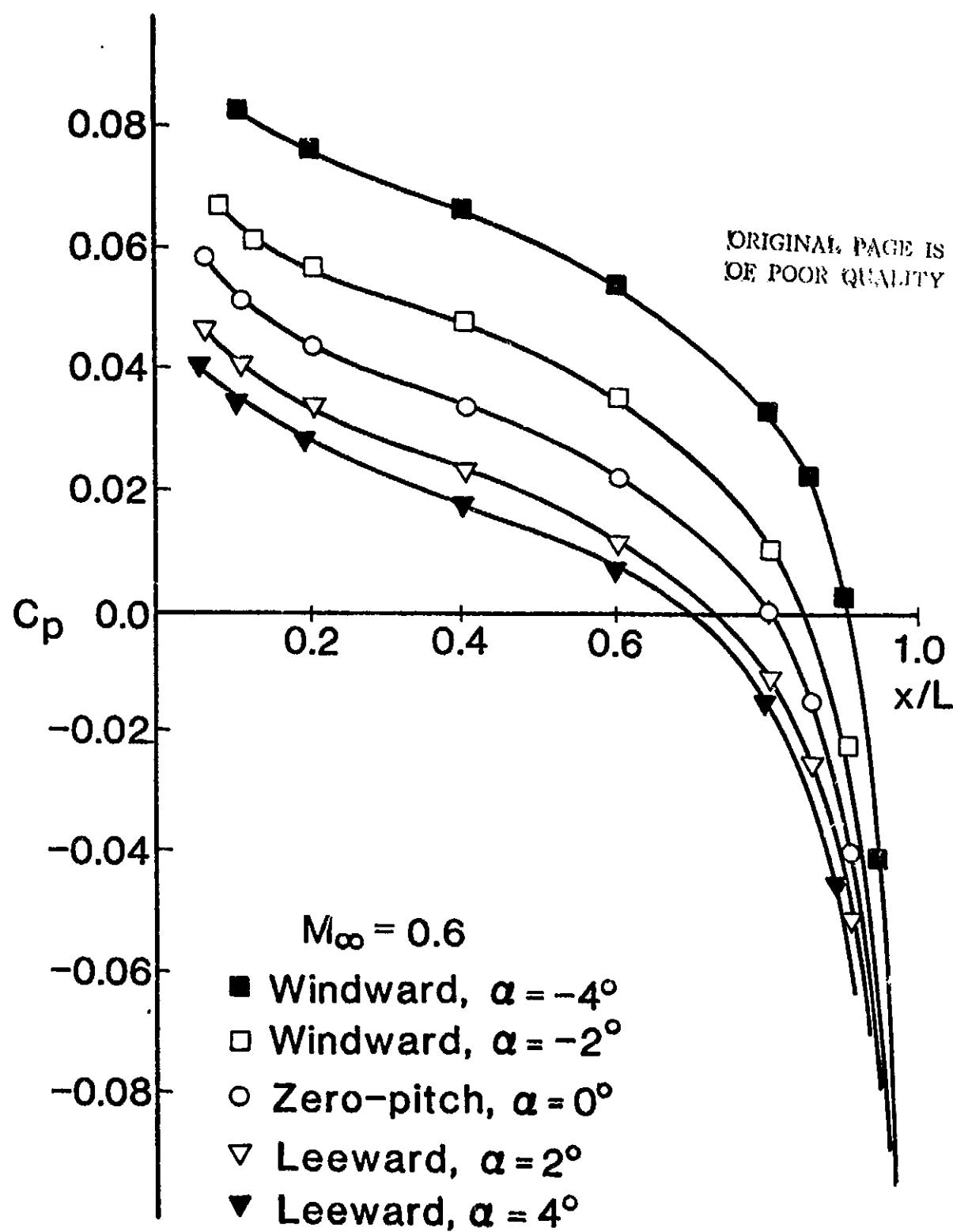


Figure 5. Calculated Pressure Coefficient on Cone Surface for Various Pitch Angles

## CHAPTER VI

### STANS COMPUTER PROGRAM

Based on the work of Patankar and Spalding [3], the STANS code was developed by Crawford and Kays [2] as an implicit, finite-difference, forward-marching integration procedure which may be used for computer simulation of boundary layers with transition. The program solves simultaneously equations for conservation of mass, momentum, stagnation enthalpy and up to five mass transfer equations.

The program uses either two-dimensional planar or axisymmetric type of coordinates so that it is possible to solve for a large variety of flows by simple manipulation of variables. This is accomplished by replacing the  $y$ -coordinate with the stream function  $\psi$ . The  $u$ -velocity component is defined by

$$u = \frac{1}{\rho r} \frac{\partial \psi}{\partial y}$$

and the momentum and energy equations become

$$\rho u \frac{\partial \psi}{\partial x} + \rho u \frac{\partial}{\partial \psi} \left[ r^2 \rho u \mu_{\text{eff}} \frac{\partial u}{\partial \psi} \right] = -g_c \frac{dp}{dx} + g_c x,$$

and  $\rho u \frac{\partial h_t}{\partial x} + u \frac{\partial}{\partial \psi} \left[ r^2 \rho u \frac{\mu_{\text{eff}}}{\Pr_{\text{eff}}} \frac{\partial h_t}{\partial \psi} \right]$

$$\frac{\partial}{\partial \psi} \left[ \frac{\mu_{\text{eff}}}{g_c} \left( 1 - \frac{1}{\Pr_{\text{eff}}} \right) r^2 \rho u \frac{\partial}{\partial \psi} \left( \frac{u^2}{2} \right) \right]$$

The stream function  $\psi$  is then normalized by using the transformation

$$\omega = \frac{\psi - \psi_I}{\psi_E - \psi_I} \quad (6.1)$$

where  $\psi_E$  and  $\psi_I$  are the stream function values on the boundary surfaces or boundary conditions.

A micro-integral method is used to obtain implicit finite-difference equations, which model the partial differential equations and may be used in a downstream, forward-marching solution scheme. The program solves laminar and turbulent boundary layers. Boundary-layer transition is based on the momentum-thickness Reynolds number criterion, which is defined a priori. The way the transition Reynolds number (RETRAN) is specified is as follows :

A very large value is assigned to RETRAN, e.g. 10000, so that the program is ensured to run wholly laminar. From the experimental data sheets obtained from NASA [1], the location of the minimum pressure is considered to be the onset of transition. At this location the corresponding value of  $Re_\theta$  in STANS output is then considered to be the correct RETRAN.

However, since we are presently concentrating only on the laminar boundary layer, a large value of RETRAN was always assigned in the input to STANS and no re-run was necessary.

Other input parameters and "flags" are required, a detailed description of which can be found in the STANS report L21. The edge velocity distribution and initial profiles for the velocity and total enthalpy across the boundary layer are required input for STANS. They are prepared by the extended Wu and Lock program. (See Chapter V).

The output of the program gives, at every incremental  $x$ , all the boundary-layer properties of interest, e.g.,  $u(y)$ ,  $u^+(y)$ ,  $y^+(y)$ ,  $C_f$ ,  $\delta$ ,  $\delta^*$ ,  $\theta$ ,  $Re_\theta$ ,  $P_w$ ,  $T(y)$ ,  $T_t(y)$ ,  $P_t(y)$ , ... etc. This information can then be used for theoretical analysis of the boundary layer.

## CHAPTER VII

### EFFECT OF FLOW ANGLES ON BOUNDARY-LAYER CALCULATIONS

As mentioned before in Chapter V, the angles of the free-stream flow will affect the boundary-layer flow.\* One of the objectives of this research was to investigate the capability of the available computer programs (Wu and Lock's and STAN5) to handle pitch and yaw angles that are a significant fraction of and to obtain some conclusions regarding the analytical tools needed to analyze such cases.

The original Wu and Lock program was modified to calculate the effective yaw angle for arbitrary combinations of yaw and pitch angles. The equations derived by Dunn et al [11] were used in subroutine ANGLES to calculate the azimuth angle of the probe, as discussed in Chapter V. It was found that the extended Wu and Lock program works well with all the cases studied.

One case was studied in some detail, viz., Case 40.547 which has the following data :

$$M_\infty = 0.6, Re_{ft} = 5 \times 10^6, q_\infty = 586 \text{ psi},$$
$$\alpha = \beta = 0.021^\circ.$$

-----

\*Transition is affected when  $\alpha/\delta$  is changed by  $\pm 5\%$ . (Reference 9).

This case was picked up as a start because of its relatively small Mach number would allow neglection of noise effect [14],[15]. The output of the program (edge velocities and initial profiles) was input to STANS. It was found that the results of STANS for this case were exactly the same as the case of zero flow angles. This was not unexpected since ( $\alpha$ ) and ( $\beta$ ) are very small in this case.

Then, the same case was repeated but with larger angles, viz.,  $\alpha=2.0$ ,  $\beta=2.0$  degrees, which places the probe  $135^\circ$  from the windward element, and  $\alpha=-2.0$ ,  $\beta=2.0$  degrees which places the probe  $45^\circ$  from the windward element. The results of these two runs, together with the original run, are shown in Figure 6.  $k$  is defined as  $2y/D$ . The plotted results agree with the observation that the pressures on the windward side are greater than those on the leeward side and that a zero-incidence flow lies in between these. However, by comparing the values of wall shear stress, at the probe azimuth angle, and the boundary-layer thicknesses  $\delta$ ,  $\delta^*$ ,  $\theta$ , the effect of  $\alpha$  and  $\beta$  is negligible as show in table II. It was, therefore, decided to confine the present stage of research to the cases of very small flow angles. A possible reason for STANS's insensitivity is its assumption of axisymmetry while real flow with large pitch and/or yaw angles will have significant cross flow, thus forming a three-dimensional flow.

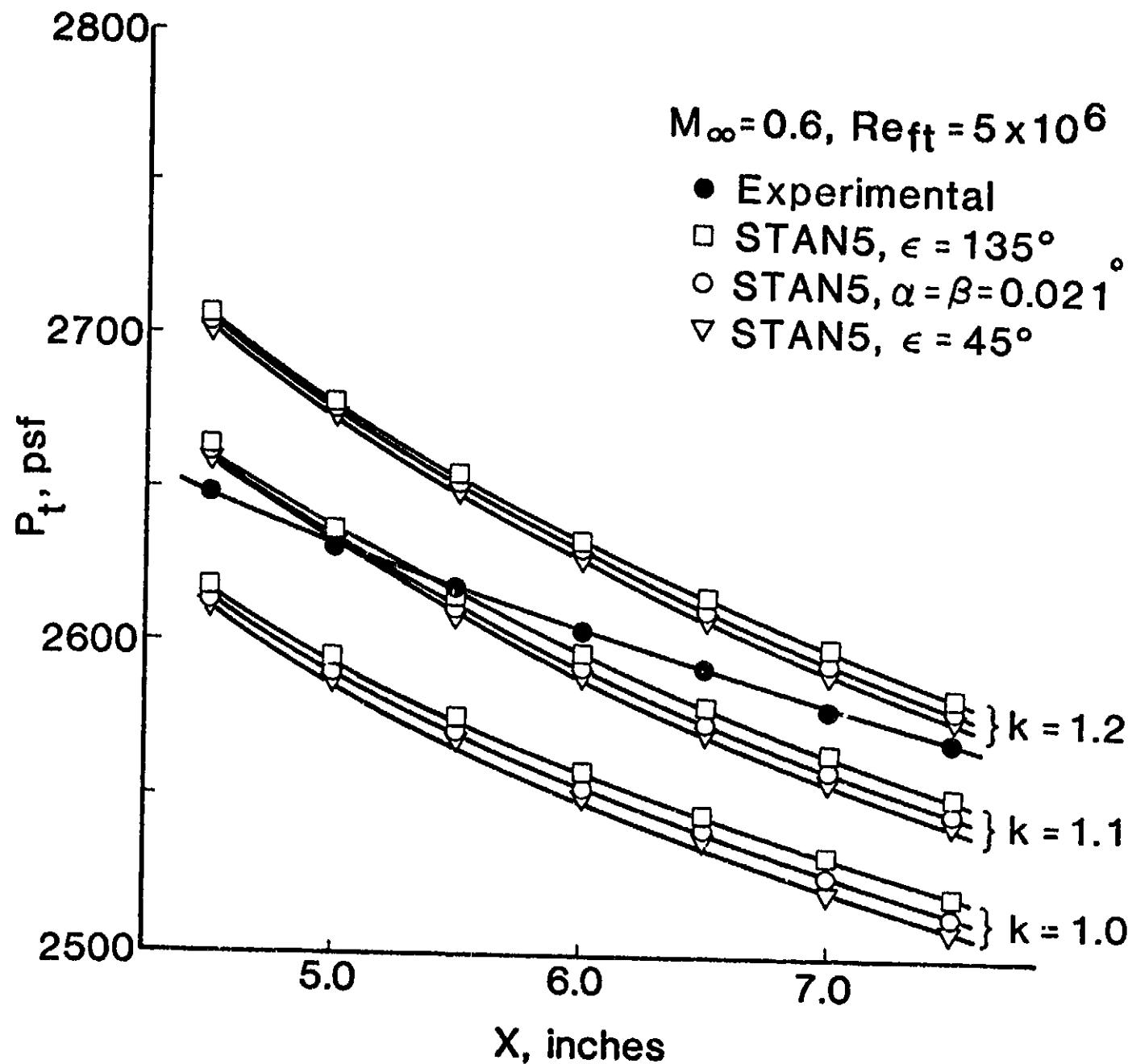


Figure 6. Theoretical Effects of Flow Angles on Effective Pressure at Different Heights in the Boundary Layer

TABLE II

SENSITIVITY OF STANS COMPUTATIONS TO  
CHANGES IN FLOW ANGLES<sup>a</sup>

x, ft	$\alpha = \beta = 0.021^\circ$				$\alpha = \beta = 2.0^\circ$ $\epsilon = 135^\circ$				$\alpha = -2.0^\circ, \beta = 2.0^\circ$ $\epsilon = 45^\circ$			
	$C_f/2$	$\delta$	$\delta^*$	$\theta$	$C_f/2$	$\delta$	$\delta^*$	$\theta$	$C_f/2$	$\delta$	$\delta^*$	$\theta$
0.1	842	615	153	56	838	572	153	56	843	574	153	56
0.2	593	811	216	78	590	849	216	78	593	853	217	78
0.3	483	999	265	95	482	1020	264	95	484	1025	265	96
0.4	417	1157	305	110	417	1182	305	110	419	1188	306	110
0.5	374	1310	341	123	372	1322	341	122	375	1330	342	123
0.6	341	1449	374	134	340	1425	374	134	342	1430	374	134

<sup>a</sup>All numbers, except x, are multiplied by  $10^6$ . The case analyzed is 40.547. The azimuthal angle,  $\epsilon$ , corresponds to the probe located at the top of the cone.

## CHAPTER VIII

### CORRELATION OF SKIN FRICTION

#### 8.1 Theoretical Background

Dimensional analysis [17] has led to a wall-law of the form

$$u^+ = f(y^+) \quad (8.1)$$

where  $u^+ = u/u^*$ ,  $y^+ = u^*y/v$  and

$u$ =longitudinal velocity,  $u^*$ =shear velocity  $= \sqrt{\tau_w/\rho}$

If we use the incompressible Bernoulli equation to relate  $u$  to the pressure difference between a Preston tube resting on the wall and local static pressure, the law-of-the-wall can then be written in the following form.

$$u_{pt} = \sqrt{2(p_{pt} - p_w)/\rho} = \sqrt{2 \Delta p / \rho} \quad (8.2)$$

$$\frac{\sqrt{2 \Delta p / \rho}}{\sqrt{\tau_w / \rho}} = f\left(\frac{u^* y_{eff}}{v}\right) \quad (8.3)$$

Now, if we further assume that the effective center of the probe,  $y_{eff}$ , is at its half-height, i.e.,  $k_{eff} = 2y_{eff}/D = 1.0$ , then

$$\frac{\Delta p}{\tau_w} = g\left(\frac{u^* D}{2v}\right) = g\left(\sqrt{\tau_w D^2 / 4 \rho v^2}\right) \quad (8.4)$$

Now multiply the numerator and denominator on the left by the appropriate factor, in order to obtain the same grouping of terms as appear in the function  $g$ .

$$\frac{\Delta p D^2 / 4 \rho v^2}{\tau_w D^2 / 4 \rho p} = g\left(\sqrt{\tau_w D^2 / 4 \rho v^2}\right)$$

or

$$\frac{\tau_w D^2}{4 \rho v^2} = F \left( \frac{\Delta P D^2}{4 \rho v^2} \right) \quad (8.5)$$

This last relation provides a convenient way for determining the skin friction since the shear stress is now uniquely related to the difference in pressure head measured with a Preston-tube static-hole combination. For a Preston tube of given geometry, the function  $F$  can theoretically be determined from pipe flow experiments where the skin friction can be deduced from measurements of pressure drop. In view of the fact that the wall-laws for pipe and boundary layer flows are identical [17], the calibration is expected to hold also in boundary layer flows.

Equation (8.5) is for incompressible flow in which the assumption of constant properties is valid. For our case, the flow is compressible and the properties, therefore, are not constant. For applications to Preston-tube data, the properties in Equation (8.5) should be evaluated at the wall (cone surface) [13], i.e.,

$$\frac{\tau_w D^2}{4 \rho_w v_w^2} = F \left( \frac{\Delta P D^2}{4 \rho_w v_w^2} \right) \quad (8.6)$$

The choice of wall properties is consistent with Bradshaw and Unsworth's correlation [20] for compressible, turbulent boundary layer.

### 8.2 Choice of the Function $F$

Patel [16], [17] established calibration curves for the laminar sublayer, buffer or transition region and

fully-turbulent layers. His correlation for the laminar sublayer is

$$y^* = 0.5 x^* + 0.037 \quad (8.7)$$

where  $x$  and  $y$  are defined, for compressible flow, as follows:

$$x^* \equiv \log_{10} \frac{\Delta P D^2}{4 \rho_w v_w^2} \quad (8.8a)$$

$$y^* \equiv \log_{10} \frac{\tau_w D^2}{4 \rho_w v_w^2} \quad (8.8b)$$

Alternatively,  $x^*$  and  $y^*$  can be expressed in the form

$$x^* = \log_{10} \left[ \frac{C_p}{8} \frac{\rho_e}{\rho_w} R_D^2 \right] \quad , \quad (8.8c)$$

$$y^* = \log_{10} \left[ \frac{C_f}{8} \frac{\rho_e}{\rho_w} R_D^2 \right] \quad . \quad (8.8d)$$

where  $C_p$  = Preston-tube pressure coefficient  
 $= (P_{pt} - P_w) / (.5 \rho_e v_e^2)$ ,

$C_f$  = local skin friction coefficient  $= \tau_w / (.5 \rho_e v_e^2)$ ,

and

$R_D$  = Preston-tube Reynolds number

It was decided to try a straight-line<sup>1</sup> correlation for laminar boundary layers in analogy with Equation (8.7), since it was expected that the behaviour of a laminar sublayer is similar to that of the laminar boundary layer.

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<sup>1</sup>Later investigation showed that using a second-order function did not improve the curve-fitting accuracy.

### 8.3 The Curve-Fitting Program

The computer program utilized for the curve fitting is called CURFIT. After applying Equation (3.1), the Preston-tube pressure data is read in, together with other parameters like  $P_w$ ,  $C_f$ ,  $\rho_w$ ,  $\nu_w$ , and  $U_e$  obtained from the Wu and Lock [4] and the STANS [2] computer programs.

The probe characteristic length,  $D$ , was first taken to be equal to the height of the probe, i.e., 0.0097" (See Figure 2). But since Patel's correlations were based on round probes, it was decided to use an equivalent diameter of the probe. This was done by assuming the probe face to be an ellipse with major and minor axes

$$2a = 0.0138 + 0.004 = 0.0178" \text{ and}$$

$$2b = 0.0097", \text{ respectively.}$$

Then the equivalent circle has an area of

$$\frac{\pi}{4} D_{eq}^2 = \pi ab \\ \therefore D_{eq} = \sqrt{4ab}$$

$$D_{eq} = 0.01314"$$

The program then calculates  $x^*$  and  $y^*$  for each observation point\*, and via a curve-fitting package prepared by Dr. Chandler [10] at Oklahoma State University, it fits the values of  $x^*$  and  $y^*$  to a straight line of the form

$$y^* = Ax^* + K \quad (8.9)$$

-----

\*Observation points were taken 0.5" apart down to the end of the laminar portion of the boundary layer.

where  $A$  (slope) and  $F$  (y-intercept) are constants to be determined by the program.

#### 8.4 Results and Improvements

The resulting straight-line fit to all points was found to be

$$y^* = 0.632 x^* + 0.415 \quad (8.10)$$

with a root-mean-square error<sup>3</sup> of 1.2%. This error was considered unsatisfactory.

It was assumed that the reason for this data scatter is the correlation model (8.9) does not account for variable property effects. These effects can be accounted for by introducing the reference temperature,  $T'$ , into the correlation. At this temperature, average values for density and viscosity can be calculated. Tetervin [18] suggested that to transfer the incompressible skin-friction relation of Ludwig and Tillman [19] into compressible form, two parameters need to be included, namely  $\bar{M}_e$  and  $T'/T_e$ . He and numerous other investigators have modeled the effects of these two parameters by introducing density and viscosity at a reference temperature. Although Allen [20] selected the reference temperature of Sommer and Short [13], we have chosen to use Eckert's formula for  $T'$  as defined in Equation (C.3). Also,

-----

<sup>3</sup> defined as

$$\left\{ \sum_{\text{all points}} \frac{\left( \frac{y^*_{\text{STAN5}} - y^*_{\text{CURFIT}}}{y^*_{\text{STAN5}}} \right)^2}{\text{No. of points}} \right\}^{1/2}$$

the use of the incompressible Bernoulli equation to calculate  $u_{Pt}$ , Equation (8.2), is not accurate. Assuming isoenergetic flow ( $T_{t,Pt} = T_{t,e}$ ) across the boundary layer,  $u_{Pt}$  can be calculated more accurately as follows:

$$P_{Pt}/P_w = (1 + \frac{\gamma-1}{2} M_{Pt}^2)^{\gamma/(\gamma-1)}$$

$$\text{or } M_{Pt}^2 = \frac{2}{\gamma-1} \left[ (P_{Pt}/P_w)^{(\gamma-1)/\gamma} - 1 \right] \quad (8.11)$$

$$\text{and } u_{Pt}/U_e = \frac{M_{Pt}}{M_e} \sqrt{\frac{T_{Pt}}{T_e}} = \frac{M_{Pt}}{M_e} \sqrt{\frac{T_{Pt}}{T_{t,Pt}}} \sqrt{\frac{T_{t,e}}{T_e}} \sqrt{\frac{T_{t,Pt}}{T_{t,e}}}$$

$$\therefore u_{Pt}/U_e = \frac{M_{Pt}}{M_e} \left( \frac{1 + \frac{\gamma-1}{2} M_e^2}{1 + \frac{\gamma-1}{2} M_{Pt}^2} \right)^{1/2} \quad (8.12)$$

and  $x^*$  is now defined as

$$x^* = \log_{10} (u_{Pt}^2 D^2 / 4 P_w^2) \quad (8.13)$$

Thus, the improved model is in the form

$$y^* = A x^* + B \log(T'/T_e) + K$$

The resulting correlation is

$$y^* = 0.655 x^* + 2.095 \log_{10}(T'/T_e) - 0.895 \quad (8.14)$$

with an rms error of 1%. To further improve the fitting accuracy, a quadratic model of the form

$$y^* = A x^{*2} + B x^* + C \log_{10}(T'/T_e) + K$$

was tried. The result is

$$y^* = 0.273 x^{*2} - 2.618 x^* + 1.645 \log_{10}(T'/T_e) + 8.921 \quad (8.15)$$

with an rms error of 0.85%. Equation (8.15) can be written in the form

$$C_f = 6.67 \times 10^9 \frac{P_w}{P_e} 10^{\log_{10}^2 (u_{Pt} D / 2 P_w) 0.546} \left( \frac{U_{Pt} D}{2 P_w} \right)^{-5.236} \left( \frac{T'}{T_e} \right)^{1.645} R_D^{-2} \quad (8.16)$$

which has an rms error of 0.85%. Figure 7 shows the data scatter in of  $C_f$ . Figure 8 compares the recommended correla-

tion (8.15) with the data. The term  $z^*$  is defined as

$$z^* = 0.273 x^{*2} - 2.618 x^* + 1.645 \log_{10}(T^*/T_e)$$

The extraneous data, which appears above the 10<sup>3</sup> line in Figure 7, corresponds to a Mach number of 0.80 and  $Re_{ft}$  of three and four million. It is speculated that these data are a result of the formation of a transonic shock on the stem of the flow-angularity probe (e.g. see Reference 8) which affects the measured values of  $P_{ref}$  and thus  $P_{pt}$ . Discarding only this particular data, a new fit results in the following equation.

$$y^* = 0.0942 x^{*2} - 0.438 x^* + 2.023 \log_{10}(T^*/T_e) + 2.272 \quad (8.17)$$

The corresponding rms error in  $C_f$  is 4.93%. Thus, Equation (8.17) is the recommended correlation for relating  $C_f$  and  $P_{pt}$  within subsonic, compressible laminar boundary layers.

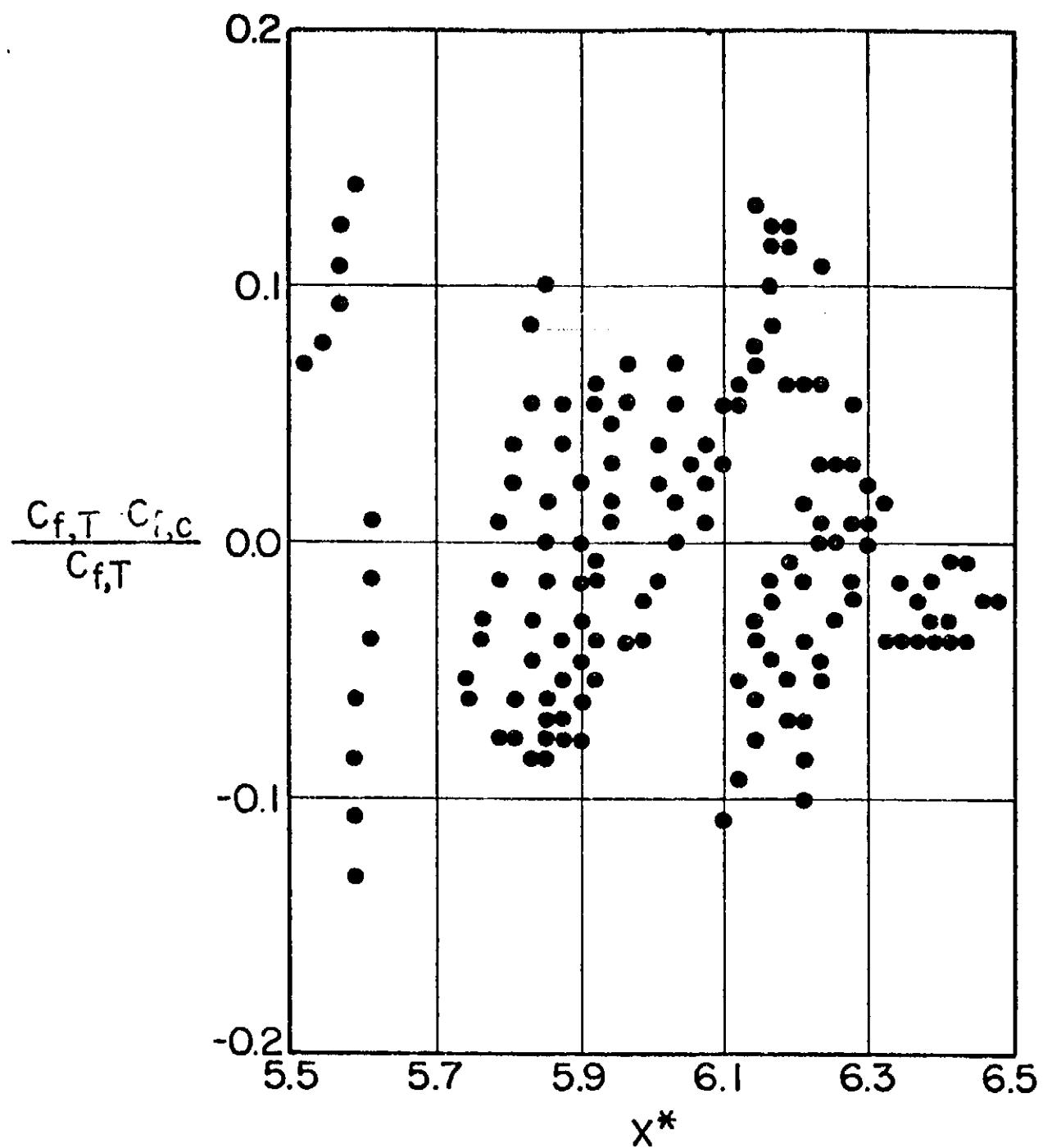


Figure 7. Deviation of Predicted Skin-Friction Coefficient by Eqn. (8.15) from Theoretical Values

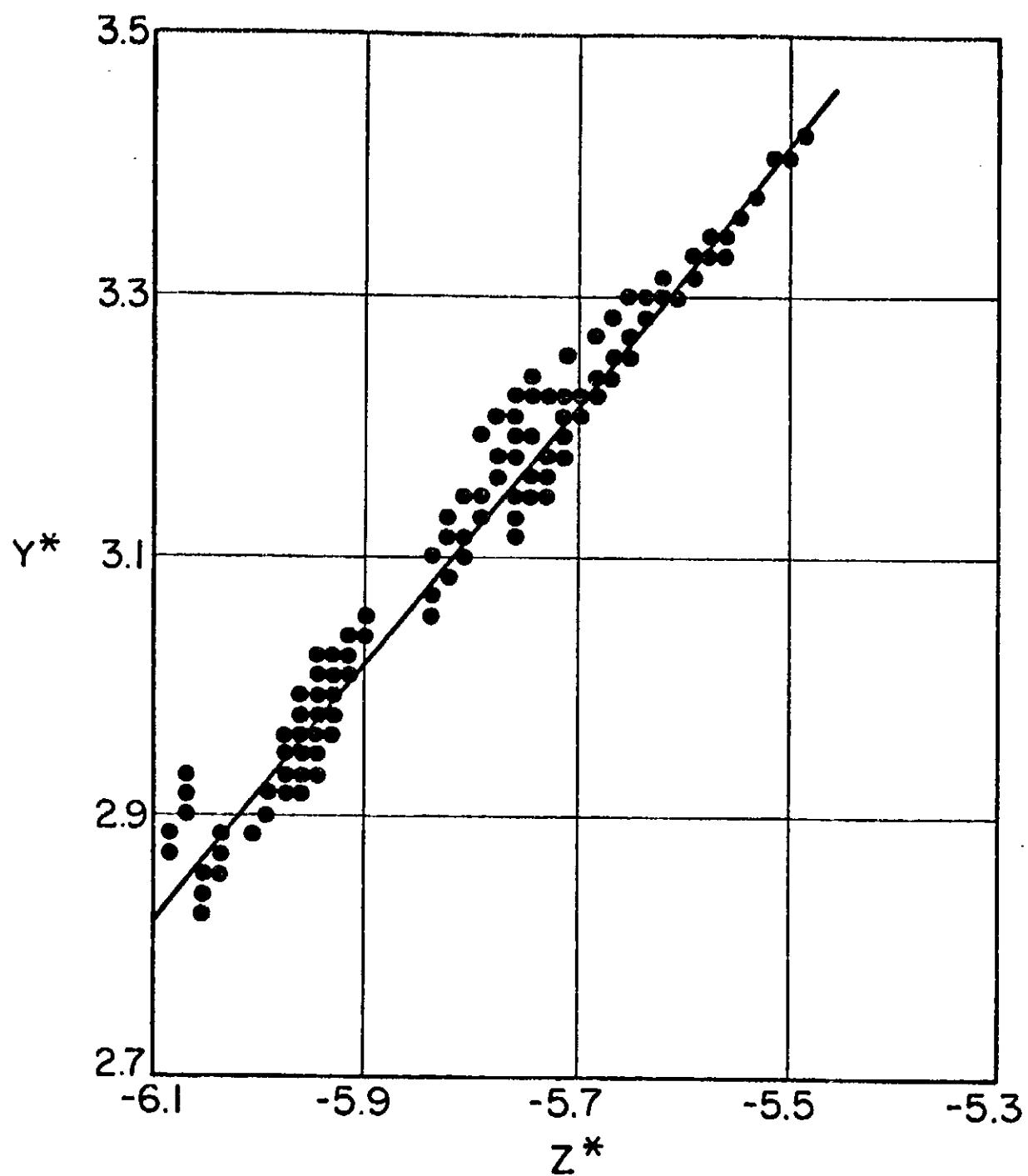


Figure 8. Data Collapse About Correlation (8.15)

## 8.5 General Remarks

a. The increase of  $D$  discussed in section 8.3, resulted in a better fit. This can be explained as follows : In the process of deriving Equation (8.4), the non-dimensional effective center of the probe,  $k_{\text{eff}} \times$  is assumed to be unity. Patel [16] and Prozorov [22] and others, e.g. Chue [17], have found that  $0.55 \leq y_{\text{eff}}/D \leq 0.65$ . Thus, writing Equation (8.4) in the form

$$\frac{\Delta P}{\tau_w} g \left[ \frac{u^* D_{\text{eq}}}{2 v} \right], D_{\text{eq}} \approx 1.3D$$

is equivalent to assuming that the average value of  $y_{\text{eff}} = 1.3 D/2 = 0.65 D$ , or equivalently  $k_{\text{eff}} = 1.3$ .

The better fit is an indication of the strong effect of the probe geometry expressed by  $R_D$ . One can also conclude that  $k_{\text{eff}}$  is a function of  $R_D$ . This conclusion was postulated before by Preston [21]. Patel [16] and Prozorov [22] showed that  $k_{\text{eff}}$  is a function of  $u_{\text{pt}} D/\nu$ .

b. Although the assumption that  $k_{\text{eff}}$  is a constant works well,  $k_{\text{eff}}$  is not a constant in fact. This can be seen in Figure 6. It increases slowly with  $x$ . It can be shown that a constant  $k_{\text{eff}}$  requires that the coefficient of  $x^*$  in the correlation be 0.5. The higher coefficient in Equation (8.10) confirms that  $k_{\text{eff}}$  is not a constant. Assuming a Blasius type profile, it is shown in Appendix D that  $k_{\text{eff}} \propto (x/D)^{.337}/(Re_x)^{.355}$ .

c. The correlation (8.15) is true for body geometries other than the cone since it is based on local variables. It

accounts for heat transfer conditions since it includes the temperature ratio  $T^*/T_e$ . It is also thought to be valid for pressure gradients since it is based on conditions near the wall. Thus, it is considered to be a general equation for estimation of skin-friction coefficients in subsonic, laminar boundary layers.

d. In an attempt to improve the curve-fit, we tried various calibration models. Among them were the following results :

$$y^* = 0.102 x^* - 0.232 \log_{10} M_\infty + 0.815 \log_{10} Re_{ft} - 0.867 \log_{10} \frac{D}{L} - 3.458 \quad (8.18a)$$

with an rms error of 0.27%,

$$y^* = 0.011 x^* - 0.582 \log_{10} (1+M_\infty^2) + 0.481 \log_{10} Re_x - 1.972 \log_{10} \frac{D}{L} - 1.554 \quad (8.18b)$$

with an rms error of 0.06%, and

$$\log_{10} \frac{1}{c_f} = 0.002 \log_{10} [(P_{Pt} - P_\infty/q_\infty) + 0.024 \log_{10} (1+M_\infty^2) + 0.501 \log_{10} (Re_{ft} D \frac{X}{L}) + 1.688] \quad (8.18c)$$

with an rms error of 0.02%. Though the accuracy of the fit became better and better, the dependence on  $x^*$  and the Preston-tube measurements became less and less. This means that STAN<sub>E</sub> calculations were correlated in these calibration models rather than the experimental data.

The use of freestream parameters ( $M_\infty, Re_{ft}$ ) in correlations (8.18a,b,c) limits their use to the 10-degree cone measurements, i.e., the coefficients of  $M_\infty$  and  $Re_{ft}$  in these correlations are not universal. To correct for that, the local Mach number  $M_e$  and  $Re_x$  should be used.

e. The calibration models used by Bradshaw and Unsworth, Allen, Fenter and Stallmach, and Patel which were reported by Allen in his survey report [20] were all tried for the present laminar data. It was found that none of them was competitive with our correlation in terms of the rms error in skin-friction coefficient. Allen's 2nd-degree model fitted the laminar data with an rms error in  $C_f$  of 8.6%.

f. Bradshaw and Unsworth [23] have criticized Allen's use of the reference temperature to evaluate density and viscosity in the classical law-of-the-wall. Rather than replace the conventional evaluation of properties at the wall, we have followed the procedure by Teterivin [18] and others to obtain a compressible equation for  $C_f$  by simply multiplying an incompressible equation for  $C_f$  by the ratio of  $T^*/T_e$  raised to some exponent. Here we have determined the exponent via a curve fit of the data. Thus, we are partially accounting for Bradshaw and Unsworth's objection. However, their second objection still applies to our analysis in that the reference temperature method is based on zero-pressure-gradient flows and has an unknown range of validity for flows with pressure gradients.

#### 8.6 Prozorov Correlation

Assuming a relatively small height of the Preston-tube, Prozorov [22] expanded the velocity  $u$  about the wall using MacLaurin's series and reached the following simple correlation for incompressible laminar boundary layers.

$$C_f = \frac{1}{q_e} \left( \frac{\mu u_{pt}}{y_{eff}} - \frac{1}{2} y_{eff} \frac{dp}{dx} \right) \quad (8.19)$$

He analytically verified Equation (8.19) for round and rectangle openings of the probe (for which  $y_{eff}$  can be theoretically calculated).

Correlation (8.19) has the advantage that it can be used for high pressure gradients\*, and the disadvantage that  $y_{eff}$  must be known a priori.

It is also limited to incompressible flows.

It is worth mentioning that Prozorov's paper is the only study found in the literature that discusses correlating Preston-tube data with theoretical laminar shear stress.

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\*All the cases investigated in this study had small favorable  $dp/dx$ .

## CHAPTER IX

### CONCLUSIONS

1. The Wu and Lock computer program is an accurate and reliable way of calculating the inviscid flow field about a sharp cone at transonic speeds.

With the added subroutines, the program is now capable of calculating the inviscid pressure and velocity distribution along a conical ray, corresponding to the Preston tube survey, for arbitrary combinations of pitch and yaw angles. It also calculates compressible initial profiles based on similarity theory and the supersonic laminar cone rule; this information is used to start the boundary layer computations.

2. The STANS computer code does not work satisfactorily when the flow angles are significant. It was found that its calculations were insensitive to changes in the flow angles when other parameters were kept the same. This limits its utility.

3. It is possible to correlate skin friction and experimental Preston-tube pressure measurements in the simple form (8.17).

4. The non-dimensional effective center of the Preston tube,  $k_{eff}$ , is not a constant value but rather increases with  $x$  and decreases with  $k_x$ .

## CHAPTER X

### SUPPLEMENTARY OBSERVATIONS

1. A 3-dimensional boundary-layer computer code is needed to continue investigation of the role of pitch and yaw angles on the correlation of Preston-tube data and skin friction.
2. The laminar correlation needs to be verified in supersonic flows and also for free-flight conditions for which the wall temperature seldom equals the adiabatic wall temperature.
3. By using the measured Preston-tube pressures at the end of transition, the correlation of Allen [20] can be used to initiate computation of the fully-developed turbulent boundary layers on the cone. This avoids tackling the development of a skin-friction correlation for the boundary-layer transition region until the laminar and turbulent correlations are established.
4. The laminar correlation may be connected with Allen's and/or Bradshaw and Unsworth's [20] correlations for turbulent boundary layers in order to model boundary-layer transition.
5. In order to verify and make use of Prozorov's [22] findings, a method is required that relates the Preston-tube pressure to the geometry of the probe. One way of doing this

is by curve-fitting the computed values of  $k_{eff}$  (obtained from plots similar to Figure 6) with  $x$  and  $Re_x$ .

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## APPENDIX A

### AZIMUTH ANGLE CALCULATION

In this appendix are presented the equations developed by Dunn et al [11] to locate the windward element. Figure 9 is a schematic of a typical vehicle at angle of attack which defines the parameters used in this appendix. As illustrated, the angles of pitch and yaw are measured with respect to the freestream velocity vector. It should be noted that the relationship utilized to determine the location of the windward element is sensitive to calculation accuracy. For this reason, double precision is used in the computer subprogram ANGLES. The pitch and yaw angles are restricted to magnitudes less than 90 degrees.

The first step is to evaluate the angle between the vehicle axis and the resolved yaw vector. This angle will be denoted by  $\phi$ .

$$\sin(\phi) = c/f$$

$$\tan(\phi) = c/d$$

$$\sin(\alpha) = c/e$$

$$\tan(\alpha) = c/a$$

$$\sin(\alpha)/\sin(\phi) = (c/e)/(c/f) = t/e$$

$$\tan(\alpha)/\tan(\phi) = (c/a)/(c/d) = d/a$$

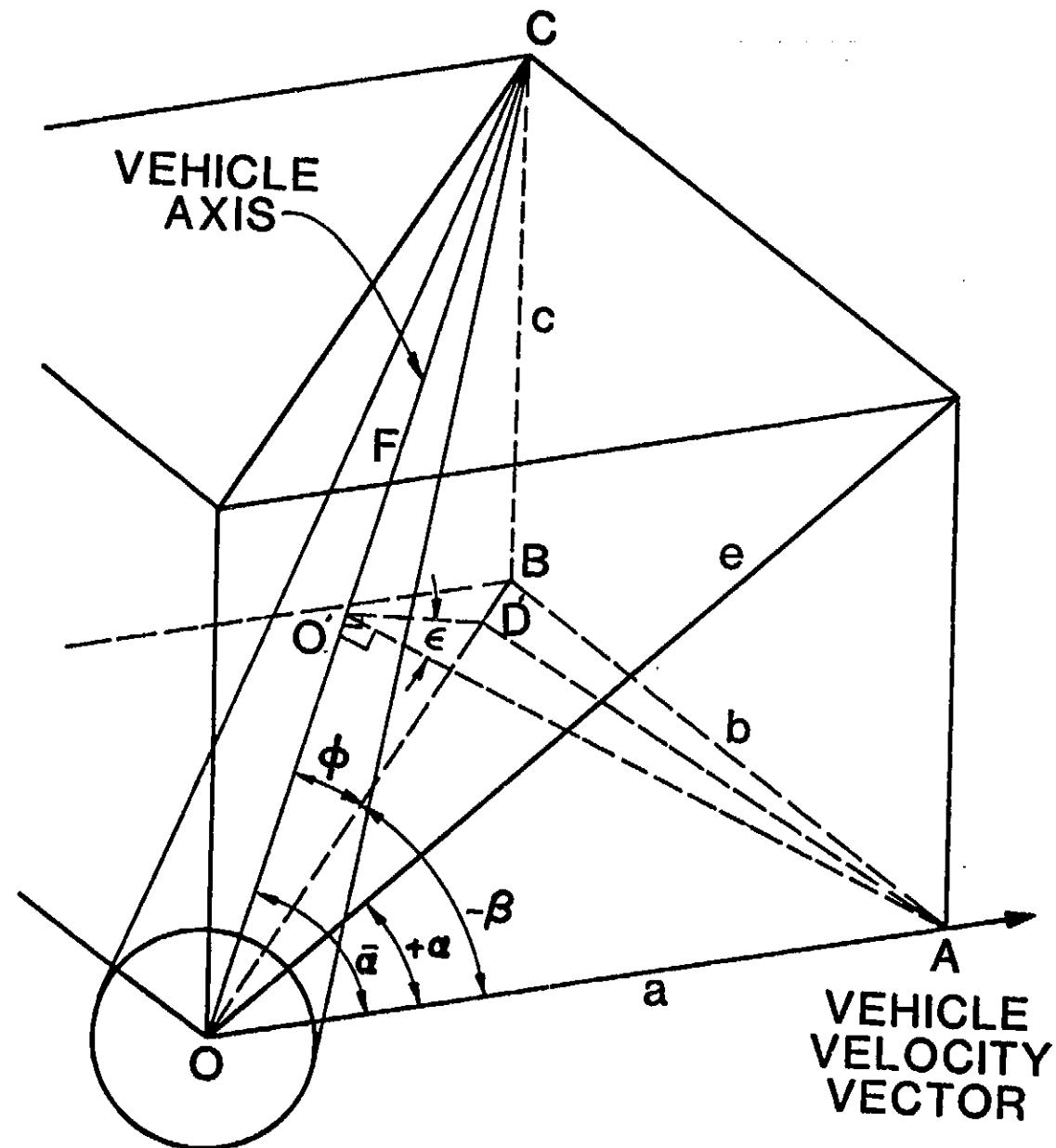


Figure 9. Schematic of Flow Angles

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$\cos(\beta) = a/d$

Thus  $\tan(\phi) = (a/d) \tan(\alpha) = \cos(\beta) \tan(\alpha)$

$$\tan(\phi) = \cos(\beta) \tan(\alpha) \quad (A.1)$$

Now the angle  $\bar{\alpha}$  can be calculated as follows :

$$d/f = \cos(\phi), \cos(\bar{\alpha}) = a/f = (a/d)(d/f)$$

$$\therefore \cos(\bar{\alpha}) = \cos(\beta) \cos(\phi), 0 < \bar{\alpha} < 90 \quad (A.2)$$

Equation (A.2) determines the angle  $\bar{\alpha}$  which is denoted as the effective angle of attack.

At this point, we want to find the angle that the windward vector makes with the vehicle axis. The following results can be obtained from Figure 9.

$$\cos(\epsilon) = (0^{\circ}D^{\circ})/(0^{\circ}A)$$

$$0^{\circ}A = a \sin(\bar{\alpha})$$

$$0^{\circ}O = a \cos(\bar{\alpha})$$

$$\tan(\phi) = (0^{\circ}D^{\circ})/(0^{\circ}O)$$

$$\cos(\epsilon) = (0^{\circ}D^{\circ})/a \sin(\bar{\alpha}) = (0^{\circ}D^{\circ}) \tan(\phi) / a \sin(\bar{\alpha})$$

$$\cos(\epsilon) = a \cos(\bar{\alpha}) \tan(\phi) / a \sin(\bar{\alpha})$$

$$\therefore \cos(\epsilon) = \cot(\bar{\alpha}) \tan(\phi) \quad (A.3)$$

an alternate expression for calculating  $\epsilon$  can be formed by substituting Equation (A.1) for  $\tan(\phi)$

$$\cos(\epsilon) = \cot(\bar{\alpha}) \cos(\beta) \tan(\alpha)$$

## APPENDIX B

### CALCULATION OF FREESTREAM PROPERTIES

Values of  $M_\infty$ ,  $q_\infty$  and  $Re_{ft}$  are specified for a given wind tunnel setting. From these values, all properties of the freestream can be calculated as follows :

1. Obtain the freestream total pressure  $P_{t,\infty}$  as follows :

$$q_\infty = \frac{1}{2} \rho_\infty U_\infty^2 = \frac{1}{2} \rho_\infty M_\infty^2 (\gamma P_\infty / \rho_\infty) \\ = \gamma M_\infty^2 P_\infty / 2 \quad (B.1)$$

$$P_{t,\infty} = \frac{P_{t,\infty}}{P_\infty} \cdot \frac{P_\infty}{q_\infty} \cdot q_\infty \\ = 2 q_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} / \gamma M_\infty^2 \quad (B.2)$$

Note that the total pressure and temperature are constant for isentropic, subsonic flow.

2. Obtain the freestream static temperature as follows :

$$\frac{M_\infty Re_{ft}}{q_\infty} = \frac{U_\infty}{\sqrt{\gamma R T_\infty}} \cdot \frac{\rho_\infty U_\infty}{\mu_\infty} \cdot \frac{2}{\rho_\infty U_\infty^2} = \frac{2}{\mu_\infty \sqrt{\gamma R T_\infty}} \\ = \frac{2 (T_\infty + 198.6)}{2.27 \times 10^{-8} T_\infty^{1.5} \sqrt{\gamma R T_\infty}} \\ \therefore \left( \frac{M_\infty Re_{ft}}{q_\infty} \cdot \frac{2.27 \times 10^{-8} \gamma R}{2} \right) T_\infty^2 - T_\infty - 198.6 = 0 \quad (B.3)$$

Where Sutherland's relation is used for  $\mu_\infty$ .

3. Using Equations (B.1) and (B.3),  $\rho_\infty$  can be obtained from the the perfect gas relation :

$$\rho_\infty = P_\infty / R T_\infty \quad (B.4)$$

4. Also the total temperature can be found using the isen-  
tropic relation :

$$T_t = T_\infty \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right) \quad (B.5)$$

This procedure is automated in subroutine DIST of the extended Wu and Lock program described in Chapter V. The listing of the routine can be found in Appendix F.

## APPENDIX C

### CALCULATION OF INITIAL PROFILES

Since STANS is a forward-marching, finite-difference program, starting profiles of velocity and total enthalpy are required to calculate subsequent velocity and total enthalpy profiles along the cone. Care should be taken, therefore, in calculating these initial profiles. However, the effect of the starting profile on the calculations becomes small after a certain developmental distance, as shown in Figure 1.

The edge velocity distribution can be expressed [13] as follows

$$U_e = c x^n \quad (C.1)$$

Where  $c$  and  $n$  are constants. Fitting Equation (C.1) to typical edge velocities near the tip of the cone, as obtained from the extended Wu and Lock program<sup>1</sup>, results in  $n = 0.0047$ .

The pressure gradient parameter for a conical flow,  $\beta$ , is related to the inviscid velocity distribution [13] by

$$\beta_c = \frac{2n}{3+n} = 0.003128.$$

This in turn corresponds to a wedge flow with

$$n_w = n_c/3 = 0.00157.$$

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<sup>1</sup>Case analyzed here was Case 40.547.

Examination of Figure 4-11 of White [13] indicates that the solution for  $f(\eta)$  corresponding to  $\beta=0$  is expected to be good. Therefore, the tabulated solution for Blasius flow may be used to specify the initial profiles. The normal distance  $y_c$  can be calculated now from

$$y_c = y_B / \sqrt{3}$$

$$\therefore y_c = \eta_B / \left[ \frac{3 U_e}{2 v' x} \right]^{1/5} \quad (C.2)$$

Where  $v'$  is the kinematic viscosity evaluated at the reference temperature  $T^*$  as will be shown now. One can obtain Equation (C.2) using Mangler transformation.

An expression for the reference temperature across the boundary layer is given by Eckert's formula [13] :

$$T' = T(0.5 + 0.039 M^2 + 0.5 T_w/T) \quad (C.3)$$

Where  $T = T_t / (1 + \frac{\gamma-1}{2} M^2)$  (C.4)

and  $T_w = T_{aw} = T(1 + r \frac{\gamma-1}{2} M^2)$  (C.5)

Where  $r = (\text{Pr})^{1/2}$  for laminar boundary layer, and  
 $= (\text{Pr})^{1/3}$  for turbulent boundary layer.

The values of local Mach numbers  $M_e$  and the total temperature  $T_t$  are calculated by the extended Wu and Lock program. Prandtl number is taken to be 0.72 for air.

Now  $\mu'$  can be calculated using Sutherland's relation

$$\mu' = 2.27 \times 10^{-8} \frac{(T')^{1.5}}{T' + 198.6} \quad (C.6)$$

To obtain  $\nu'$ ,  $\rho'$  is calculated using the perfect gas relation

$$\rho' = P_w / R T' \quad (C.7)$$

Where the static pressure  $P_w$  is calculated in the main Wu and Lock program.

Therefore, from (C.6) and (C.7),  $\nu'$  can be calculated,

$$\nu' = \mu'/\rho' \quad (C.8)$$

Substitution in (C.2), yields a table of  $y_c$  vs.  $\eta_B$ . From the Blasius solution for  $t^* = u/U_e$ , we can obtain a table of  $u$  vs.  $\eta_B$ . Thus the initial velocity profile is specified.

The total enthalpy is defined as

$$h_t = h + \frac{u^2}{2 g_c J}$$

$$= c_p T + \frac{u^2}{2 g_c J} \quad \text{for } c_p = \text{constant (C.9)}$$

The distribution of  $T$  through the boundary layer may be approximately expressed [13] as

$$T \approx T_w + (T_{aw} - T_w) \frac{u}{U_e} - \frac{r u^2}{2 c_p g_c J}$$

Substitution into (C.9) gives

$$h_t = c_p [T_w + (T_{aw} - T_w) \frac{u}{U_e}] + \frac{(1-r) u^2}{2 g_c J}$$

With the assumption  $T_w = T_{aw}$ , this equation reduces to

$$h_t = c_p T_w + \frac{(1-r) u^2}{2 g_c J} \quad (C.10)$$

$T_w$  was calculated via the Wu and Lock program, and the initial stagnation profile was defined by Equation (C.10).

## APPENDIX D

### FUNCTIONAL DEPENDENCE OF THE EFFECTIVE CENTER OF THE PROBE

For simplicity, we will derive an expression for  $k_{\text{eff}}$  for incompressible flow over a flat plate. The correlation (8.14) reduces in this case to

$$C_f \sim C_p^{0.655} R_D^{0.69} \quad (D.1)$$

where  $C_f = \tau_w / (0.5 \rho u^2)$ ,

$$C_p = \Delta P / (0.5 \rho u^2),$$

$$R_D = U D / v = (U x / v) D / x = Re_x D / x. \quad \text{Since } \Delta P = \frac{1}{2} \rho u_{pt}^2$$

$C_p$  can be written in the form

$$C_p = (u_{pt} / U)^2 = (f^*)^2 \text{ at } \eta_{\text{eff}}$$

where  $f^*$  = 1st derivative of the Blasius function w.r.t.  $\eta$ , and

$$\eta_{\text{eff}} = y_{\text{eff}} \sqrt{U/2 v x} \sim k_{\text{eff}} D \sqrt{Re_x} / x.$$

Since the height of the probe is very small (0.0097"), all the laminar boundary-layer data was obtained within the lower 40% of the layer thickness. In this region  $f^* \sim \eta$  is valid.

Therefore,  $C_p$  can be expressed as

$$C_p \sim \eta_{\text{eff}}^2 \sim k_{\text{eff}}^2 D^2 Re_x / x^2. \quad (D.2)$$

Substituting relation (D.2) into (D.1) gives

$$C_f \sim k_{\text{eff}}^{1.31} Re_x^{-0.035} \left(\frac{x}{D}\right)^{-0.441} \quad (D.3)$$

The well-known relation for  $C_f$  in this case is

$$C_f \sim Re_x^{-0.5} \quad (D.4)$$

Comparing (D.3) and (D.4), the following equation is obtained for  $k_{eff}$ .

$$k_{eff} \sim Re_x^{-0.355} \left(\frac{x}{D}\right)^{0.337} \quad (D.5a)$$

$$\text{Or alternatively, } k_{eff} \sim Re_x^{-0.018} R_D^{-0.337} \quad (D.5b)$$

Again, relations (D.5a,b) are only valid for incompressible flow over a flat plate and are presented here only to demonstrate that  $y_{eff}$  is not a constant.

APPENDIX E  
RAW DATA USED FOR SKIN-FRICTION  
CORRELATION

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POINT NO.	X INF	Y INF	Z INF	X	DELTA P	P INF	UE	RH CW	RH CE	NUW	NUE	THEIA	CF			
1	3.63	3.52E	3.7	585.3	4.5	323.6	2325.0	636.3	3.084127	0.086915	0.14414E-03	0.13046E-03	0.0001065			
2	3.63	2.51E	3.7	585.3	5.5	326.5	2325.0	636.6	0.084103	0.089899	0.14419E-03	0.13047E-03	0.0001123			
3	3.63	2.51E	3.7	586.0	5.5	263.7	2225.0	636.9	0.084154	0.086883	0.14422E-03	0.13045E-03	0.0001174			
4	3.63	2.52E	3.7	586.0	6.3	279.4	2325.0	637.2	0.094558	0.088869	0.14427E-03	0.13050E-03	0.0001228			
5	3.63	2.52E	3.7	586.0	6.5	268.0	2325.0	637.5	0.084554	0.088855	0.14426E-03	0.13051E-03	0.0001278			
6	3.63	2.52E	3.7	586.0	7.3	255.2	2225.0	637.8	0.064334	0.088814	0.14429E-03	0.13052E-03	0.0001324			
7	3.63	2.52E	3.7	586.0	7.5	246.6	2225.0	638.1	0.084321	0.088829	0.14431E-03	0.13054E-03	0.0001370			
8	3.63	2.52E	3.7	586.0	7.5	246.6	2225.0	638.1	0.084321	0.088829	0.14431E-03	0.13054E-03	0.0001370			
9	3.63	2.52E	3.7	586.0	7.5	277.0	4.5	279.4	0.067518	0.071424	0.18135E-03	0.16422E-03	0.0001123			
10	3.63	2.52E	3.7	586.0	7.5	279.4	1893.0	640.5	0.067564	0.071410	2.18143E-03	0.16422E-03	0.0001191			
11	3.63	2.52E	3.7	586.0	7.5	277.0	4.5	252.3	1893.0	640.9	0.067546	0.071397	2.18148E-03	0.16424E-03	0.0001253	
12	3.63	2.52E	3.7	586.0	7.5	277.0	5.5	238.1	641.3	0.067531	0.071384	2.18152E-03	0.16426E-03	0.0001314		
13	3.63	2.52E	3.7	586.0	7.5	277.0	6.3	228.1	1893.0	641.6	0.067510	0.071373	0.18159E-03	0.16428E-03	0.0001370	
14	3.63	2.52E	3.7	586.0	7.5	277.0	6.5	221.0	1893.0	641.9	0.067509	0.071361	0.18157E-03	0.16433E-03	0.0001427	
15	3.63	2.52E	3.7	586.0	7.5	277.0	7.3	215.3	1893.0	642.1	0.067495	0.071350	0.18159E-03	0.16431E-03	0.0001483	
16	3.63	2.52E	3.7	586.0	7.5	277.0	7.5	205.6	1893.0	642.4	0.067483	0.071340	0.18163E-03	0.16433E-03	0.0001533	
17	3.63	2.52E	3.7	586.0	7.5	277.0	8.3	203.9	1893.0	642.7	0.067470	0.071330	0.18167E-03	0.16435E-03	0.0001584	
18	3.63	2.52E	3.7	586.0	7.5	277.0	9.0	139.7	1417.0	541.5	0.050595	0.053491	0.24196E-03	0.21885E-03	0.0001652	
19	3.63	2.52E	3.7	586.0	7.5	277.0	9.5	136.5	1417.0	541.8	0.050584	0.053483	0.24232E-03	0.21892E-03	0.0001713	
20	3.63	2.52E	3.7	586.0	7.5	277.0	10.0	129.7	1417.0	542.1	0.050573	0.053475	0.24237E-03	0.21894E-03	0.0001773	
21	3.63	2.52E	3.7	586.0	7.5	277.0	10.5	126.5	1417.0	542.3	0.050566	0.053469	0.24210E-03	0.21896E-03	0.0001825	
22	3.63	2.52E	3.7	586.0	7.5	277.0	11.0	124.0	1417.0	542.8	0.050552	0.053454	0.24214E-03	0.21898E-03	0.0001885	
23	3.63	2.52E	3.7	586.0	7.5	277.0	11.5	121.2	1417.0	543.0	0.050542	0.053447	0.24219E-03	0.21903E-03	0.0001936	
24	3.63	2.52E	3.7	586.0	7.5	277.0	12.0	118.3	1417.0	543.3	0.050536	0.053440	0.24219E-03	0.21904E-03	0.0001974	
25	3.63	2.52E	3.7	586.0	7.5	277.0	12.5	116.9	1417.0	543.5	0.050526	0.053433	0.24216E-03	0.21906E-03	0.0002089	
26	3.63	2.52E	3.7	586.0	7.5	277.0	13.0	112.6	1417.0	542.8	0.050526	0.053427	0.24223E-03	0.21907E-03	0.0002137	
27	3.63	2.52E	3.7	586.0	7.5	277.0	13.5	104.0	1417.0	537.8	0.082484	0.085747	0.14803E-03	0.13802E-03	0.0001259	
28	3.63	2.52E	3.7	586.0	7.5	277.0	14.0	94.4	5.5	215.3	0.082471	0.085737	0.14803E-03	0.13803E-03	0.0001268	
29	3.63	2.52E	3.7	586.0	7.5	277.0	14.5	84.0	5.5	230.5	1.0	0.082461	0.14834E-03	0.13803E-03	0.0001377	
30	3.63	2.52E	3.7	586.0	7.5	277.0	15.0	74.2	6.0	193.9	2309.0	0.082458	0.085718	0.14803E-03	0.13805E-03	0.0001433
31	3.63	2.52E	3.7	586.0	7.5	277.0	15.5	54.8	6.0	250.5	5.5	0.082451	0.085706	0.14804E-03	0.13806E-03	0.0001483
32	3.63	2.52E	3.7	586.0	7.5	277.0	16.0	44.9	7.5	188.2	5.5	0.082454	0.085699	0.14807E-03	0.13807E-03	0.0001536
33	3.63	2.52E	3.7	586.0	7.5	277.0	16.5	34.9	7.5	230.9	5.5	0.082456	0.085692	0.14808E-03	0.13808E-03	0.0001586
34	3.63	2.52E	3.7	586.0	7.5	277.0	17.0	24.0	8.0	178.2	230.9	0.082453	0.085684	0.14810E-03	0.13809E-03	0.0001634
35	3.63	2.52E	3.7	586.0	7.5	277.0	17.5	13.2	6.5	171.1	2309.0	0.082457	0.085677	0.14811E-03	0.13810E-03	0.0001672
36	3.63	2.52E	3.7	586.0	7.5	277.0	18.0	6.3	172.6	5.5	0.082459	0.085670	0.14812E-03	0.13811E-03	0.0001654	
37	3.63	2.52E	3.7	586.0	7.5	277.0	18.5	5.5	122.4	5.5	0.061786	0.0864239	0.19717E-03	0.18382E-03	0.0001714	
38	3.63	2.52E	3.7	586.0	7.5	277.0	19.0	3.2	32.1	0.061775	0.064232	0.19719E-03	0.18384E-03	0.0001714		

PRINT NO.	MINF	REFT	CINF	X	DELTAP	PINF	UE	RHOF	VUF	NUE	THETA	CF	
43	2.53	2.30E 37	302.0	7.5	116.5	1726.0	526.4	0.061745	0.197226	0.18385E-33	0.0301773	0.0308866	
44	2.53	2.31E 37	302.0	8.3	115.5	1726.0	538.7	0.061767	0.197215	0.18386E-33	0.0301833	0.0308386	
45	2.53	2.31E 37	302.0	8.5	112.6	1726.0	538.9	0.061763	0.197224	0.18388E-33	0.0301868	0.030814	
46	2.53	2.31E 37	302.0	9.0	111.2	1726.0	525.1	0.061753	0.197257	0.18389E-33	0.0301944	0.030790	
47	2.53	2.31E 37	302.0	9.5	108.3	1726.0	539.3	0.061748	0.197260	0.1839CE-33	0.0301996	0.030768	
48	2.53	2.32E 37	302.0	12.3	105.5	1726.0	525.5	0.061744	0.197277	0.18391E-33	0.0302046	0.030748	
49	2.53	2.32E 37	302.0	12.5	102.6	1726.0	525.7	0.061735	0.197285	0.18392E-33	0.0302096	0.030732	
50	2.53	2.32E 37	302.0	11.0	101.2	1726.0	539.9	0.061735	0.197285	0.18393E-33	0.0302144	0.030714	
51	2.53	2.33E 37	302.0	11.5	106.4	1726.0	542.1	0.061726	0.197300	0.18394E-33	0.0302191	0.030700	
52	2.53	2.41E 37	549.0	4.5	366.5	1598.0	742.4	0.056807	0.19705E-33	0.1901190	0.0300964		
53	2.53	2.41E 37	548.0	5.0	285.1	1598.0	742.4	0.056785	0.19705E-33	0.1901190	0.0300964		
54	2.53	2.41E 37	548.0	5.5	266.6	1598.0	742.1	0.056767	0.19705E-33	0.19012CE-33	0.0301252	0.0300914	
55	2.70	0.4CE 37	548.0	6.0	256.6	1598.0	743.5	0.056744	0.19015E-33	0.19015E-33	0.0301309	0.0300874	
56	2.70	2.4CE 37	548.0	6.5	246.6	1598.0	743.5	0.061153	0.19762E-33	0.19318E-33	0.0301371	0.0300834	
57	2.70	2.4CE 37	548.0	7.0	238.1	1598.0	744.2	0.056738	0.19759E-33	0.19720E-33	0.0301428	0.0300800	
58	2.70	2.4CE 37	548.0	7.5	230.9	1598.0	744.5	0.056723	0.19765E-33	0.19022E-33	0.0301479	0.0300772	
59	2.70	2.4CE 37	548.0	8.0	223.6	1598.0	744.8	0.056692	0.19758E-33	0.19025E-33	0.0301529	0.0300746	
60	2.70	2.4CE 37	548.0	8.5	365.5	1593.0	740.0	0.076907	0.19777E-33	0.19328E-33	0.03001580	0.0300722	
61	2.70	2.5CE 37	687.0	5.2	343.6	1583.0	740.6	0.070882	0.17328E-03	0.15157E-03	0.0301061	0.0300862	
62	2.70	2.5CE 37	682.0	5.5	329.3	1483.0	741.0	0.070854	0.17636E-03	0.15160E-03	0.0301119	0.0300816	
63	2.70	2.5CE 37	680.0	6.0	313.6	1983.0	741.3	0.070827	0.17350E-03	0.15162E-03	0.0301173	0.0300780	
64	2.70	2.5CE 37	682.0	6.5	296.5	1593.0	741.7	0.070618	0.17631E-03	0.15164E-03	0.03001224	0.0300746	
65	2.70	2.5CE 37	680.0	7.0	199.0	1593.0	741.1	0.042554	0.15157E-03	0.15167E-03	0.03003273	0.0300716	
66	2.70	2.5CE 37	698.0	6.0	155.4	1190.0	741.5	0.042489	0.045795	0.28924E-03	0.25276E-03	0.0301518	0.0301008
67	2.70	2.5CE 37	698.0	6.5	151.1	1190.0	741.8	0.042480	0.045785	0.28929E-03	0.25275E-03	0.0301582	0.0300966
68	2.70	2.5CE 37	748.0	7.0	146.8	1190.0	742.1	0.042470	0.045776	0.28938E-03	0.25283E-03	0.0301650	0.0300924
69	2.70	2.5CE 37	648.0	7.5	142.4	1190.0	742.4	0.042457	0.045767	0.28943E-03	0.25286E-03	0.0301710	0.0300692
70	2.70	2.5CE 37	408.0	8.0	128.3	1190.0	743.0	0.042446	0.045759	0.28951E-03	0.25289E-03	0.0301768	0.03005860
71	2.70	2.5CE 37	409.0	8.5	135.4	1190.0	743.0	0.042437	0.045751	0.28957E-03	0.25292E-03	0.0301623	0.0300836
72	2.70	2.7CE 37	408.0	9.0	132.6	1190.0	743.3	0.042429	0.045742	0.28959E-03	0.25295E-03	0.0301879	0.0300610
73	2.70	2.7CE 37	408.0	9.5	128.3	1190.0	742.6	0.042423	0.045735	0.28962E-03	0.25298E-03	0.0301935	0.0300786
74	2.70	2.7CE 37	408.0	10.0	125.5	1190.0	743.8	0.042411	0.045728	0.28971E-03	0.25301E-03	0.0302036	0.0300748
75	2.70	2.7CE 37	409.0	10.5	122.4	1190.0	744.1	0.042402	0.045720	0.28977E-03	0.25304E-03	0.0302086	0.0300733
76	2.49	2.4CE 37	403.0	6.5	162.5	3598.0	470.4	0.109310	0.12605E-03	0.12611E-03	0.0301430	0.0300866	
77	2.49	2.4CE 37	403.0	7.0	156.8	3598.0	473.6	0.109300	0.12602E-03	0.12607E-03	0.0301486	0.0300776	
78	2.49	2.4CE 37	403.0	7.5	152.5	3598.0	470.8	0.109250	0.12602E-03	0.12607E-03	0.0301535	0.0300750	
79	2.49	2.4CE 37	403.0	8.0	149.7	3598.0	471.0	0.109280	0.12603E-03	0.12606E-03	0.0301586	0.0300726	

POINT NO.	REFI	CINF	X	DELTAP	PINF	UE	RHOM	RHOE	NUM	AUE	THETA	CF
86	2.45	1.42E-37	493.0	6.5	145.4	3558.0	471.2	6.165280	0.112000	0.126025E-03	2.0001634	0.000704
87	2.45	2.35E-37	246.0	6.5	132.0	2196.0	433.6	0.078472	0.084454	0.15514E-03	0.0001657	0.000930
88	2.45	1.32E-37	246.0	7.0	101.2	2156.0	433.8	0.078467	0.084449	0.15515E-03	0.14832E-03	0.0001716
89	2.45	2.33E-37	246.0	7.5	96.4	2156.0	434.0	0.078463	0.084446	0.15515E-03	0.14834E-03	0.0001775
90	2.45	2.33E-37	246.0	7.5	96.9	2196.0	434.0	0.078455	0.084438	0.15517E-03	0.14835E-03	0.0001834
91	2.45	2.33E-37	246.0	6.5	95.5	2196.0	434.3	0.078460	0.084434	0.15514E-03	0.14835E-03	0.0001886
92	2.45	2.32E-37	246.0	5.5	94.1	2196.0	434.5	0.078451	0.084425	0.15517E-03	0.14836E-03	0.0001945
93	2.45	2.32E-37	246.0	5.5	92.7	2196.0	434.7	0.078444	0.084424	0.15518E-03	0.14836E-03	0.0001996
94	2.45	2.31E-37	246.0	13.0	85.6	2156.0	434.9	0.078437	0.084415	0.15519E-03	0.14837E-03	0.0002048
95	2.45	2.31E-37	246.0	13.5	86.4	2196.0	435.0	0.078434	0.084415	0.15519E-03	0.14838E-03	0.0002099
96	2.45	2.31E-37	246.0	11.5	85.5	2196.0	435.0	0.078438	0.084411	0.15517E-03	0.14838E-03	0.0002143
97	2.45	2.31E-37	230.0	6.5	55.5	3651.0	316.8	0.138750	0.146880	0.83372E-04	0.081335E-04	0.0001436
98	2.45	2.31E-37	230.0	7.0	89.8	3651.0	317.3	0.138740	0.146880	0.83379E-04	0.081337E-04	0.0001489
99	2.45	2.31E-37	230.0	7.5	85.5	3651.0	317.1	0.138740	0.146870	0.83380E-04	0.081339E-04	0.0001535
100	2.45	2.31E-37	230.0	8.0	82.7	3651.0	317.2	0.138740	0.146870	0.83386E-04	0.081341E-04	0.0001589
101	2.45	2.31E-37	230.0	8.5	78.4	3651.0	317.4	0.138740	0.146860	0.83377E-04	0.081343E-04	0.0001638
102	2.45E-37	398.0	4.0	230.0	9.0	3536.0	564.6	0.074434	0.076235	0.246932E-03	0.23974E-03	0.0001420
103	2.45E-37	396.0	4.5	222.4	3536.0	564.9	0.074428	0.076232	0.24734E-03	0.23977E-03	0.0001508	
104	2.45E-37	396.0	5.0	213.8	3536.0	565.5	0.074420	0.076226	0.24736E-03	0.23977E-03	0.0001587	
105	2.45E-37	396.0	5.5	202.9	3536.0	565.5	0.074417	0.076220	0.24742E-03	0.23977E-03	0.0001662	
106	2.45E-37	396.0	6.0	196.7	3536.0	565.7	0.074401	0.076215	0.24743E-03	0.23980E-03	0.0001738	
107	2.45E-37	396.0	6.5	185.6	3536.0	566.0	0.074400	0.076210	0.24742E-03	0.23981E-03	0.0001808	
108	2.45E-37	396.0	7.0	161.1	3536.0	566.2	0.074398	0.076205	0.24642E-03	0.23682E-03	0.0001878	
109	2.45E-37	396.0	7.5	219.5	1559.0	737.6	0.056558	0.069337	0.21574E-03	0.18867E-03	0.0001425	
110	2.45E-37	396.0	7.5	211.0	1565.0	738.0	0.056545	0.069326	0.21575E-03	0.18869E-03	0.0001476	
111	2.45E-37	396.0	7.5	1569.0	728.3	0.056532	0.069150	0.21583E-03	0.18872E-03	0.0001530		
112	2.45E-37	396.0	8.0	159.6	1565.0	738.0	0.056518	0.069150	0.21589E-03	0.18874E-03	0.0001579	
113	2.45E-37	396.0	8.5	156.2	1565.0	738.0	0.056510	0.069150	0.21594E-03	0.18876E-03	0.0001628	
114	2.45E-37	396.0	9.0	1377.0	842.4	0.048737	0.053626	0.25553E-03	0.21522E-03	0.0001310		
115	2.45E-37	396.0	9.5	1377.0	842.8	0.048714	0.053611	0.25567E-03	0.21524E-03	0.0001368		
116	2.45E-37	396.0	10.0	1377.0	843.1	0.048698	0.053598	0.25575E-03	0.21527E-03	0.0001423		
117	2.45E-37	396.0	10.5	1377.0	843.4	0.048686	0.053586	0.25580E-03	0.21531E-03	0.0001477		
118	2.45E-37	396.0	11.0	1377.0	843.7	0.048672	0.053574	0.25587E-03	0.21534E-03	0.0001527		
119	2.45E-37	396.0	11.5	1377.0	844.0	0.048656	0.053562	0.25595E-03	0.21537E-03	0.0001575		
120	2.45E-37	396.0	12.0	1350.0	937.1	0.048462	0.053343	0.25397E-03	0.21371E-03	0.0001475		
121	2.45E-37	396.0	12.5	1350.0	937.4	0.048443	0.053331	0.25407E-03	0.21374E-03	0.0001527		
122	2.45E-37	396.0	13.0	1350.0	937.7	0.048428	0.053328	0.25414E-03	0.21377E-03	0.0001576		

POINT NO.	MIN	MAX	REF-T	QINF	X	DELTAP	PINF	UE	RHCM	RHCE	NUE	NUW	THETA	CF	
117	3.89	4.42E-07	605.3	8.5	208.1	1350.0	928.0	0.25418E-03	0.21380E-03	0.0001623	0.0000700				
118	3.93	4.51E-07	761.3	4.5	416.2	1695.0	937.2	0.066866	0.20315E-03	0.17121E-03	0.0001061	0.0000860			
119	3.92	4.52E-07	761.0	5.0	354.5	1695.0	937.7	0.660737	0.20329E-03	0.17125E-03	0.0001118	0.0000814			
120	3.93	4.52E-07	761.0	5.5	373.5	1699.0	938.0	0.066827	0.20340E-03	0.17128E-03	0.0001171	0.0000776			
121	3.93	4.52E-07	761.0	6.0	355.0	1695.0	938.4	0.660710	0.20350E-03	0.17131E-03	0.0001223	0.0000744			
122	3.93	4.52E-07	761.0	6.5	336.4	1699.0	928.8	0.060679	0.20356E-03	0.17134E-03	0.0001273	0.0000714			
123	3.93	4.32E-07	453.7	8.2	159.6	1211.0	837.3	0.036300	0.339573	0.338735E-03	0.0001821	0.0000832			
124	3.93	4.32E-07	453.2	8.5	106.5	1211.0	837.6	0.036265	0.339565	0.338855E-03	0.0001875	0.0000808			
125	3.93	4.32E-07	453.0	9.0	105.5	1211.0	837.8	0.036263	0.339557	0.338870E-03	0.0001929	0.0000786			
126	3.93	4.32E-07	453.0	9.5	102.6	1211.0	838.1	0.036276	0.339949	0.338910E-03	0.0001982	0.0000764			
127	3.93	4.32E-07	453.0	10.0	99.6	1211.0	838.4	0.036265	0.339021E-03	0.285010E-03	0.0002035	0.0000744			
128	3.93	4.32E-07	453.0	10.5	96.5	1211.0	838.6	0.036259	0.339934	0.339066E-03	0.0002084	0.0000728			
129	3.93	4.32E-07	492.0	8.5	134.3	867.0	927.6	0.3131438	0.035437	0.393655E-03	0.315070E-03	0.0001871	0.0000826		
130	3.93	4.32E-07	492.0	9.0	132.6	867.0	927.9	0.031427	0.335425	0.390780E-03	0.315120E-03	0.0001925	0.0000784		
131	3.93	4.32E-07	492.0	9.5	131.2	867.0	928.2	0.031415	0.335420	0.290950E-03	0.315180E-03	0.0001975	0.0000762		
132	3.93	4.32E-07	492.0	10.0	125.7	867.0	928.5	0.031406	0.035541	0.391030E-03	0.315220E-03	0.0002032	0.0000742		
133	3.93	4.32E-07	492.0	10.5	128.2	867.0	928.8	0.031400	0.035494	0.391060E-03	0.315270E-03	0.0002078	0.0000724		
134	3.93	4.32E-07	492.0	11.0	128.3	867.0	929.0	0.031390	0.035397	0.391200E-03	0.315320E-03	0.0002127	0.0000708		
135	3.93	4.32E-07	492.0	11.5	128.6	867.0	929.3	0.031362	0.035386	0.391320E-03	0.315370E-03	0.0002175	0.0000692		
136	3.93	4.32E-07	842.0	5.0	359.2	1485.0	934.4	0.052921	0.235650E-03	0.193660E-03	0.0001115	0.0000812			
137	3.93	4.32E-07	842.0	5.5	372.5	1485.0	934.9	0.052884	0.059532	0.235850E-03	0.193710E-03	0.0001166	0.0000774		
138	3.93	4.32E-07	842.0	6.0	355.0	1485.0	925.3	0.052855	0.059511	0.236000E-03	0.190750E-03	0.0001219	0.0000742		
139	3.93	4.32E-07	842.0	6.5	335.0	1485.0	935.7	0.059492	0.236070E-03	0.193750E-03	0.0001269	0.0000712			
140	3.93	4.32E-07	842.0	7.0	345.0	1485.0	934.4	0.052921	0.235650E-03	0.193660E-03	0.0001115	0.0000812			
141	3.93	4.32E-07	842.0	7.5	340.7	1485.0	974.2	0.035621	0.45075	0.313130E-03	0.248370E-03	0.0001184	0.0000956		
142	3.93	4.32E-07	691.0	5.0	322.2	1397.0	974.8	0.039483	0.039532	0.248440E-03	0.301247	0.0000908			
143	3.93	4.32E-07	691.0	5.5	305.1	1397.0	975.2	0.039551	0.045040	0.313860E-03	0.248510E-03	0.0001305	0.0000866		
144	3.93	4.32E-07	691.0	6.0	290.8	1297.0	975.6	0.039523	0.045024	0.314130E-03	0.248500E-03	0.0001362	0.0000828		
145	3.93	4.32E-07	691.0	6.5	275.4	1097.0	977.4	0.039500	0.312740E-03	0.248390E-03	0.0001115	0.001016			
146	3.93	4.32E-07	691.0	7.0	270.5	1097.0	977.0	0.039498	0.045020	0.314400E-03	0.248620E-03	0.0001420	0.0000794		
147	3.93	4.32E-07	691.0	7.5	262.3	1097.0	976.7	0.039463	0.044595	0.314540E-03	0.248640E-03	0.0001472	0.0000766		
148	3.93	4.32E-07	691.0	8.0	253.8	1097.0	977.1	0.039447	0.044981	0.314650E-03	0.248720E-03	0.0001525	0.0000740		
149	3.93	4.32E-07	691.0	8.5	243.8	1097.0	977.4	0.039431	0.044957	0.314730E-03	0.248770E-03	0.0001571	0.0000718		
150	3.93	4.32E-07	691.0	9.0	394.9	1382.0	977.0	0.039415	0.056415	0.251490E-03	0.248820E-03	0.0001619	0.0000696		
151	3.93	4.32E-07	873.0	5.5	365.2	1282.0	976.3	0.039526	0.056394	0.251720E-03	0.199340E-03	0.0001115	0.0000810		
152	3.93	4.32E-07	873.0	6.0	349.3	1382.0	978.7	0.049490	0.056374	0.251950E-03	0.199440E-03	0.0001168	0.0000774		
153	3.93	4.32E-07	873.0	6.5	330.7	1382.0	979.1	0.049467	0.056356	0.252070E-03	0.199450E-03	0.0001267	0.0000712		

APPENDIX F  
LISTING OF THE EXTENDED WU AND LOCK  
PROGRAM WITH AN EXAMPLE RUN

```

C
C
C      EXTENDED WU & LOCK PROGRAM.
C
C
C      MAIN PROGRAM ( ORIGINAL WU AND LOCK PROGRAM SLIGHTLY MODIFIED ).*
C
C
C      THIS PROGRAM CALCULATES THE PRESSURE COEFFICIENT OVER A CONE IN
C      TRANSONIC FLOW AT AN ANGLE OF ATTACK, FOR WHICH SHOCK IS DETACHED.
C      INPUT PARAMETERS ARE EXPLAINED INSIDE THE PROGRAM.
C
C
      REAL MINF, XSTS, MAX
      COMMON /MSTR/ PT, GAMMA, GAI, G42, GCMH
      COMMON /INF/ PINF, TINF, VINF
      COMMON /SUG/ PIOT, PI(125), V(125), X(125), Z(125)
      COMMON /PARM/ NPT, ISTART, MAXIT, TOL, TTOT, XL, IPASS, DELTA
      DOUBLE PRECISION TARIKH
      DIMENSION Z(125), F(125), FN(125), D(125), Y(125), CP(125),
     1 U(125), SE(125), SF(125), FN3(125), CO(125),
     2 ZL(125), BL(125), CL(125), DL(125), EU(125), FL(125), SC(125)
      DIMENSION TITLE(125)
      NAMELIST /RCUP1/ MINF, ALFA, BETA
      NAMELIST /RCUP2/ FEFT, QINF
      READ (5,*), NPT, ISTART, MAXIT, TOL, TTOT, XL, IPASS, DELTA
      C
      PINF = FREE-STREAM MACH NUMBER,
      DELTA = SEMI-VERTEX ANGLE OF CONE IN DEGREES,
      BEJA = YAW ANGLE IN DEGREES,
      ALFA = ANGLE OF ATTACK IN DEGREES,
      REFL = REYNOLDS NUMBER PER FCCT, AND
      DINF = FREE-STREAM DYNAMIC PRESSURE.
      C
      CALL DATE (TARIKH)
      WRITE E '6,30) TARIKH
      30      FORMAT 1/* DATE RUN : * ,AB/
      26      READIS, 770, END=20)TITLE
      770      FORMAT(20A4)
      20      WRITE(6,780)TITLE
      WRITE(7,770)TITLE
      780      FORMAT(1X,20A4/)
      READ (5,30)P1
      C
      THREE=1./3.
      C
      DO C000010
      00 000013
      00 000015
      00 000016
      00 000017
      00 000018
      00 000020
      00 000025
      00 000030
      00 000033
      00 000037
      00 000040
      00 000042
      C
      DO C000043
      00 000044
      00 000045
      00 000046
      00 000047
      00 000048
      00 000049
      00 000050
      00 000060
      C
      DO C000070
      C

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00 0000680
00 0000090
00 0000100
00 0001100
00 0001200
00 0001300
00 0001400
00 0001500
00 0001600
00 0001700
00 0001800
00 0001900
00 0002000
00 0002100
00 0002200
00 0002300
00 0002400
00 0002500
00 0002600
00 0002700
00 0002800
00 0002900
00 0003000
00 0003100
00 0003200
00 0003300
00 0003400
00 0003500
00 0003600
00 0003700
00 0003800
00 0003900
00 0004000
00 0004100
00 0004200
00 0004300
00 0004400
00 0004500
00 0004600
00 0004700
00 0004800
00 0004900
00 0005000
00 0005100

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ORIGINAL IMAGE IS  
OF POOR QUALITY

```

C FIRST ITERATION :
C
C KSIS=.98
C
C NPI=NPI-1
C
C DO 4 I=2,121
C A=A+.01
C X(I)=93.11-21*3
C Y(I)=4
C F(I)=X(I)+S15*(1.+A*(C1*A*(C2+C3*A(I)))
C
C 4 CONTINUE
C
C A=0.0
C H=X-L*C
C F111=0.0
C
C 5 CONTINUE
C
C H=E-H
C
C 6 J=1,101
C A=-0.01
C
C 7 J=1,101
C
C 8 CONTINUE
C
C 9 I=2,NPI
C
C 10 CONTINUE
C
C 11 SE(I)=F(I)/S0RT1(Y(I))-A1**2*(X(I)-DE1/XS15)**2
C
C 12 CONTINUE
C
C 13 F111=0.0
C F1K=Z(NPI)*G1M1*BE*F(I)*SCTG*F(I)
C IF F1K.GT.0.0 IFN(I)=F1NK**THIRD
C IF FN(I).LT.0.0 FN(I)=F(I)*0.5
C FN(I)=FN(I)-F(I)
C U(I)=F(I)-(FN(I)**2)/GAM
C
C 14 CONTINUE
C
C 15 CALL SEK181H,SE,Z,NPI)
C F111=0.0
C F1K=Z(NPI)*G1M1*BE*F(I)*SCTG*F(I)
C IF F1K.GT.0.0 IFN(I)=F1NK**THIRD
C IF FN(I).LT.0.0 FN(I)=F(I)*0.5
C FN(I)=FN(I)-F(I)
C U(I)=F(I)-(FN(I)**2)/GAM
C
C 16 CONTINUE
C
C 17 CALL SEK181H,FN,Z,NPI)
C KSIS=1.0/Z(NPI)
C
C 18 DO 19 I=2,NPI
C CALL SEK181H,FN,Z,I)
C X(I)=KSIS*Z(I)
C
C 19 CONTINUE
C
C 20 DO 21 I=1,NPI
C B1F=ABS(D(I))
C

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```

      CT(I)=FN(I)-F(I)
      U(I)=(Z(I)-(Y(I))**2)/GAM
      CONTINUE
C
C     CALL SEKIB(H,FN,Z,RPT)
      KSIS=1./Z(NPT)
      00 0001200
      00 0001210
      00 0001220
C
C     DO 80 I=2,NPT
      CALL SEKIB(H,FN,Z,RPT)
      X(I)=KSIS*Z(I)
      00 0001230
      00 0001240
C
C     CONTINUE
      00 0001250
      00 0001260
      00 0001270
      00 0001280
C
C     ITER=ITER+1
      00 0001290
C
C     DO 81 I=1,NP1
      F(I)=FN(I)
      CP(I)= -2.*Z(I)-DEL**2*(1.-4.*((SIN(YETA))**2)*(SIN(YETA))**2)
      PTL=(I+5*GAMMA*CP(I)*MINF**2)/PT*GA2
      ZZ=PTL**(-1./GA2)-1.)*2./GA1
      ZZ=AMAX1(ZZ,ZERO)
      Z(I)=SQRT(ZZ)
      00 0001300
      00 0001310
      00 0001320
      00 0001330
      00 0001340
      00 0001350
      00 0001360
      00 0001370
C
C     CONTINUE
      00 0001380
      00 0001390
      00 0001400
      00 0001410
C
C     IF(IITER-3)70,301,301
      00 0001420
      00 0001430
      00 0001440
      00 0001450
      00 0001460
      00 0001470
      00 0001480
C
C     30 101 I=2,NPT
      IF(X(I)-99)100,101,101
      BARF=ABS(D(I))
      00 0001490
      00 0001500
      00 0001510
      00 0001520
      00 0001530
C
C     CONTINUE
      00 0001540
C
C     103 IF(10AD-1)1102,104,104
      104 IF(IITER-25)73,2,2
      102 CONTINUE
      ITER=ITER+ITER
C
      WRITE(16,26)ITER,KSIS
C

```

```

111 IF(IALFA .LE. 310, 311, 310
112   DO 300 I=1,NPT
113   CD(I,I)=CP(I,I)*X(I,I)*KSIS*FN(I,I)
114   CONTINUE
115   CALL SEKIS(IH,CD,I,Z,NPT)
116   CD=2.*Z(NPT)
117
118   WRITE(6,801)CD
119   CONTINUE
120
121   GCN=5.0*GAMMA*MINF**2
122   HUNKY=.5/TAN(IDE/L)
123   WRITE(6,227)
124   DO 32 K=1,NPT
125   Z=X(K)*HUNKY
126   PI(K)=CP(K)*GCN**1.0
127   WRITE(6,228) K,X(K),CP(K),Z4(K),P(K),K
128   CONTINUE
129
130   FORMAT(10X,'ANGLE OF ATTACK=',F10.3,5X,'ANGLE OF YAW=',F10.3,
131   2,1DEGEE,SJ '')
132   2 FORMAT(10X,'SUPERSONIC FLOW = NO CONVERGENCE '///)
133   25 FORMAT(10X,12,' ITERATIONS ',5X,'KSAT(SONIC) * ',F15.5/)
134   26 FORMAT(10X,12,' X/L ',T41,'CP',T57,'M',T70,'P/PINF',/)
135   227 FORMAT(115,4F15.5,11G)
136   228 FORMAT(10X,' MACH INF = ',F8.4,5X,'CONE SEMI-VERTEX ANGLE = '
137   625   ' ,F9.6,'1DEG)',5X,'GAMMA = ',F8.3/)
138   229 FORMAT(10X,' DRAG COEFFICIENT = ',F13.5/)
139   801 IF (IPASS.EQ.2) GOTO 96
140   READ (5,301)P2
141   CALL DISI (4INF,REFT,QINF)
142   IF (IPASS.EQ.1) GO TO 95
143   CALL INITIA
144   WRITE (7,95)
145   95 FORMAT(90*'')
146   96 WRITE(6,94)
147   94 FORMAT(1H)
148   20 STOP
149   END

```

```
C
C      SUBROUTINE SEK1B
C      ****
C
C      SUBROUTINE SEK1B(H,W,Z,NDIM)
C      DIMENSION H(1125),W(1125)
C      SUM2=0.0
C      DO 1 I=2,NDIM
C      SUM1=0.0
C      SUM2=SUM2+H(I)*W(I)+W(I-1)
C      1   Z(I-1)=SUM1
C      Z(NDIM)=SUM2
C
C      RETURN
C
C      END
```



```

C
C
C SUBROUTINE DIST
C
C THIS SUBPROGRAM CALCULATES THE FREE-STREAM CONDITIONS AS WELL AS
C THE VELOCITY DISTRIBUTION AT THE EDGE OF THE ECLIPSY LAYER TO BE
C USED AS BOUNDARY CONDITIONS TO A BOUNDARY LAYER SOLVING PROGRAM.
C
C
C SUBROUTINE DIST (IINF,REFT,CINF)
C
C DIMENSION T(125)
C
C COMMON /MSRA/ PT, GAMMA, GA1, GA2, GCHN
C COMMON /SUE/ FTCT, PI125, V125, X125, ZM125
C COMMON /INF/ PINF, TINF, VINF
C COMMON /PARM/ NPT, ISTART, XL, IPASS, DELTA
C
C ZM'S = LOCAL MACH NUMBERS.
C
C REAL PINF, TINF
C
C PINF = FREE-STREAM DYNAMIC VISCOSITY LBF S/F 10-2.
C
C E=1.7573E+C6
C =3.5654E+C8
A=REFT*MINF/CINF
CNS1=GAMMA*1716*C
FTCT=CINF*PT*GA2/CCPM
C
C FTCT = FREE-STREAM TOTAL PRESSURE PSF.
C
C
C IINF=(R+SORT18+4.0*A*C)/2.0/A
C TCT=TINF*PT
C
C TINF = FREE-STREAM STATIC TEMPERATURE RANKIN.
C TCT = TOTAL TEMPERATURE.
C
C PINF=2.27E-C8*TINF*SQRT(TINF)/ITINF+158.6
VINF=SCRT(CCST*TINF)*TINF
PINF=PTOT/PT+C2
F=CINF*PINF/52.35/TINF
C

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81      WRITE(6,100) PINF, TINF, VINF, RHOINF, MUINF
100      FCMPAT(5X,G10.4)
101      EC 1C 1=1,NPT
102      T11=TTOT/(1.0+3.54G1*L1)**2
103      V11=L11**SRT(FCAST*T11)
C      T'S = LOCAL STATIC TEMPERATURES FANKIN.
C      V'S = EDGE VELOCITIES F/S.
C
104      CCAT TRUE
105      WRITE(6,93)
106      SFORMAT//45X,'VELOCITY DISTRIBUTION :',/
107      WRITE(6,250)
108      FORMAT//5X,'Y',13X,'X/L',13X,'U'
109      WRITE(6,200)1,2M(11),X(11),V(11),I=1,NP1)
110      FCMPAT(5X,15.5X,F10.5,5X,F10.5,5X,F10.2 )
111      FEILAN
112      WRITE(6,300)
113      FFORMAT/5X,'MAX. ITERATIONS EXCEEDED',/
114      RETURN
115      END

C      VINF = FREE-STREAM VELOCITY.
C      RHOINF = FREE-STREAM DENSITY 1BM/F1**3.
C      PINF = FREE-STREAM STATIC PRESSURE PS/F.
C
116      T11=TINF*PT
117      WRITE(6,150)
118      FORMAT//7X,'MINF',12X,'REFT',11X,'QINF',/
119      WRITE(6,110)MINF,REFT,QINF
120      FORMAT(5X,G10.4)
121      WRITE(6,160)
122      FORMAT//5X,'PTC1AL',9X,'PTC1AL',/
123      WRITE(6,120) PTC1,PTC1
124      FORMAT(5X,G10.4)
125      WRITE(6,170)
126      FORMAT//6X,'PINF',11X,'TINF',11X,'UINF',12X,'RHOCINF',10X,'MUINF',/00002510

```

```

C
C SUBROUTINE INITIA
C *****
C
C THIS SUBPROGRAM CALCULATES INITIAL VELOCITY AND TOTAL ENTHALPY
C PROFILES IN THE BOUNDARY LAYER NEAR THE TIP OF THE CONE WHICH IS
C NEEDED IN STAN-5 PROGRAM.
C
C SUBROUTINE INITIA
COMMON /SUB/ PTOI,P(125),V(125),X(125),ZM(125)
COMMON /1INF/ PINF,TINF,VINF
COMMON /PARM/ NPT,1START,T,MAXIT,TCL,TTOT,XL,IPASS,DELTA
DIMENSION ETA(30),PRIME(30),RM(125)
REAL MIN,NUSTAR,NUSTAR
DATA PR/0.77/
DATA 0002750
C
C BLASIUS SOLUTION.
C
C DATA ETA/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,C,8,C,9,1,0,1,1,2,1,3/
C DATA FPRIME/0.0,0.04656,0.09391,0.14081,C.18761,C.23423,0.28058,
C.32653,0.37196,0.41672,0.46063,0.50354,0.54525,0.58559,0.62439,
C.66147,0.69670,0.72593,0.76106,0.79000,0.81669,0.86330,0.90107,
C.93060,0.95288,0.96505,0.98757,0.99594,C.98822,1.0/
C
C READ 15,1) XLE
READ 15,1) XLE
C
C END OF LAMINAR PORTION, INCHES.
C
C XLE=XLE/12.0/XL
XLE=0
DXLE=0
IF 1 XLE.LT.X(1) 1 GO TO 57
DO 61 I=1,NPT
IF (XLE.EQ.X(I)).OR.(XLE.GT.X(I)).AND.XLE.LT.X(I+1)) GO TO 64
61 CONTINUE
IF 1 XLE.EQ.0 1
IXLE=1
XIN=X(1START)*XL
MIN=Z(1START)
UE=V(1START)
R=SQRT(P)
C
C R = RECOVERY FACTOR, PR = PRANDTL NUMBER.
C

```

```

T=11.11/(1.0+0.2*MIN**2)          00 002 820
T=1.1*(1.0+R*0.2*MIN**2)          00 002 830

C   TW = WALL TEMPERATURE = ADIABATIC WALL TEMPERATURE APPROXIMATELY.

C   TSTAR=T*(3.5+0.039*MIN**2+0.5*TWT)
MUSTAR=2.27E-08*TSTAR*SQR(TSTAR)/TSTAR+198.6)      00 002 840
      00 002 855

C   THESE ARE THE AVERAGE TEMPERATURE AND DYNAMIC VISCOSITY ACROSS
C   THE BOUNDARY LAYER AT THE INITIAL LOCATION.

C   PIN=PIN(START)*PINF          00 002 860

C   INITIAL STATIC PRESSURE.

C   WRITE(6,90)
90   FORMAT(3X,'TW',10X,'PINIT',/)
      WRITE(6,90) TW, PIN
      FORMAT(5X,'F8.2,6X,F8.2//')
      WRITE(7,70) PIN
      FORMAT(F10.2)
      CC 6 2 1=START,IXLE
      XI(1)=XL
      K(1)=X(1)*XL
      K(1)=X(1)*SINIDELTA/57.29578
      00 002 940

C   R*'S ARE THE RADII OF THE CONE SURFACE AT THE VARIOUS X*'S.

C   WRITE(7,50) X(1),R(1)
50   FORMAT(2(5X,F10.6))
      CONTINUE
      20 40 I=START,IXLE
      WRITE(E17.3Q)V(I)
      FORMAT(5X,F10.2)
      30
      C   THE EDGE VELOCITIES.

C   40  CONTINUE
      ROSTAR=PIN/53.35/TSTAR
      MUSTAR=MUSTAR/RNSTAR*32.174
      YFACT=SQR(1.0*UE/2.0/NUSTAR/XIN)
      HFACT=5.075*6/(1.0-R)
      HIN=2.24*TW
      WRITE(6,100)
      FORMAT(10X,'INITIAL PROFILE : //10X,'Y',16X,'U',14X,'H//')
      100
      DO 20 I=1,30
      20

```

```

Y=ETAL11/YFACT
U=UE*FPRI*E11
H=HIN+U*2/HFACT

C   Y = THE NORMAL DISTANCE FROM CONE SURFACE IN FEET, U & H ARE THE
C   VELOCITY AND STAGNATION ENTHALPY PROFILES RESPECTIVELY.

C
C   WRITE(6,10) Y,U,H
C   WRITE(7,10) Y,U,H
C   FORMAT(5X, E10.4,2(5X,F10.2))
C10  CONTINUE
C20  RETURN
C67  WRITE(6,69) XLE
C69  FORMAT(5X, 'ERROR IN XLE. VALUE READ =',2X,FB .5)
C80  RETURN
C90  END

```

21	0.23636	0.04341	0.48853	1.00760
22	0.21448	0.04263	0.48868	1.00749
23	0.22460	0.04226	0.48883	1.00739
24	0.23471	0.04170	0.48898	1.00730
25	0.24483	0.04116	0.48913	1.00720
26	0.25496	0.04062	0.48927	1.00711
27	0.26505	0.04009	0.48941	1.00701
28	0.27517	0.03556	0.48955	1.00652
29	0.28528	0.03904	0.48969	1.00683
30	0.29539	0.03853	0.48982	1.00674
31	0.30549	0.03801	0.48996	1.00665
32	0.31560	0.03750	0.49009	1.00656
33	0.32571	0.03700	0.49021	1.00647
34	0.33581	0.03649	0.49037	1.00638
35	0.34592	0.03598	0.49050	1.00630
36	0.35602	0.03547	0.49063	1.00621
37	0.36612	0.03497	0.49077	1.00612
38	0.37622	0.03446	0.49091	1.00603
39	0.38632	0.03394	0.49104	1.00594
40	0.39642	0.03343	0.49118	1.00585
41	0.40652	0.03291	0.49131	1.00576
42	0.41662	0.03238	0.49145	1.00567
43	0.42671	0.03186	0.49160	1.00557
44	0.43681	0.03132	0.49174	1.00548
45	0.44690	0.03078	0.49188	1.00539
46	0.45700	0.03024	0.49202	1.00529
47	0.46709	0.02968	0.49217	1.00515
48	0.47718	0.02912	0.49232	1.00510
49	0.48727	0.02855	0.49247	1.00500
50	0.49736	0.02796	0.49263	1.00489
51	0.50744	0.02737	0.49278	1.00479
52	0.51753	0.02676	0.49294	1.00468
53	0.52761	0.02615	0.49311	1.00457
54	0.53770	0.02551	0.49328	1.00446
55	0.54778	0.02487	0.49344	1.00435
56	0.55786	0.02420	0.49362	1.00424
57	0.56794	0.02352	0.49380	1.00412
58	0.57802	0.02282	0.49399	1.00399
59	0.58809	0.02210	0.49418	1.00387
60	0.59817	0.02136	0.49437	1.00374
61	0.60824	0.02059	0.49457	1.00360
62	0.61832	0.01980	0.49476	1.00346
63	0.62839	0.01898	0.49500	1.00332
64	0.63846	0.01913	0.49522	1.00317
65	0.64852	0.01725	0.49546	1.00302
66	0.65459	0.01633	0.49570	1.00286

67	0.65865	0.31537	0.49595	1.00269	67
69	0.67071	0.01437	0.49622	1.00251	68
69	0.63877	0.01332	0.49649	1.00233	69
70	0.65893	0.01223	0.49678	1.00214	70
71	0.70839	0.01107	0.49708	1.00194	71
72	0.71894	0.00996	0.49741	1.00172	72
73	0.72699	0.00857	0.49775	1.00150	73
74	0.73954	0.00721	0.49810	1.00126	74
75	0.74938	0.00576	0.49848	1.00101	75
76	0.75912	0.00422	0.49889	1.00074	76
77	0.76916	0.00259	0.49932	1.00045	77
78	0.77920	0.00281	0.49979	1.00014	78
79	0.78923	-0.00109	0.50028	0.99581	79
80	0.79926	-0.00315	0.50082	0.9945	80
81	0.80928	-0.00538	0.50141	0.99516	81
82	0.81930	-0.00782	0.50205	0.99863	82
83	0.82931	-0.01050	0.50275	0.99816	83
84	0.83932	-0.01345	0.50352	0.99765	84
85	0.84932	-0.01674	0.50438	0.99707	85
85	0.85932	-0.02042	0.50534	0.99643	86
87	0.86931	-0.02459	0.50643	0.99570	87
88	0.87928	-0.02934	0.50766	0.99497	88
89	0.88925	-0.03482	0.50909	0.99391	89
90	0.89921	-0.04124	0.51076	0.99278	90
91	0.90915	-0.04886	0.51273	0.99145	91
92	0.91907	-0.05810	0.51512	0.98953	92
93	0.92898	-0.06955	0.51807	0.98783	93
94	0.93885	-0.08416	0.52162	0.98527	94
95	0.94870	-0.10352	0.52677	0.98168	95
96	0.95849	-0.13054	0.53364	0.97716	96
97	0.96822	-0.17107	0.54386	0.9706	97
98	0.97703	-0.23915	0.56082	0.95815	98
99	0.98722	-0.37904	0.59455	0.93367	99
100	0.99592	-0.84974	0.70507	0.85130	100
101	1.00000	-2.5C762	1.09189	0.56117	101

$W_{INF}$	$q_{E/FT}$	$Q_{INF}$
-500.0	-4003E+37	404.0
PICTAL		
2733.0	530.8	
PIRF		
2309.0	505.5	
$T_{INF}$	$T_{INF}$	$R_{HCINF}$
		$W_{INF}$
	551.1	-85559 E-01
		-3666 E-06

VELOCITY DISTRIBUTION :

	$W$	$X/L$	$U$
1	0.0	0.0	0.0
2	0.48140	0.01181	531.45
3	0.48255	0.02197	533.13
4	0.48389	0.03213	534.08
5	0.48452	0.04228	534.75
6	0.48522	0.05242	535.27
7	0.48543	0.06257	535.70
8	0.48578	0.07271	536.07
9	0.48605	0.08284	536.40
10	0.48637	0.09298	536.70
11	0.48663	0.10311	536.97
12	0.48687	0.11325	537.22
13	0.48709	0.12338	537.45
14	0.48730	0.13350	537.67
15	0.48749	0.14362	537.88
16	0.48769	0.15376	538.08
17	0.48787	0.16398	538.27
18	0.48804	0.17400	538.45

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19	0.48820	0.18412	538.62
20	0.48831	0.15424	538.80
21	0.48853	0.20436	538.96
22	0.48868	0.21449	539.13
23	0.48883	0.22460	539.29
24	0.48898	0.23471	539.44
25	0.48913	0.24483	539.60
26	0.48927	0.25494	539.74
27	0.48941	0.26505	539.89
28	0.48955	0.27517	540.04
29	0.48969	0.28526	540.19
30	0.48982	0.29539	540.33
31	0.48996	0.30549	540.47
32	0.49009	0.31560	540.62
33	0.49023	0.32571	540.76
34	0.49037	0.33581	540.90
35	0.49050	0.34592	541.04
36	0.49063	0.35602	541.18
37	0.49077	0.36612	541.33
38	0.49091	0.37622	541.47
39	0.49104	0.38632	541.61
40	0.49118	0.39642	541.76
41	0.49131	0.40652	541.90
42	0.49145	0.41662	542.05
43	0.49160	0.42671	542.20
44	0.49174	0.43681	542.34
45	0.49188	0.44690	542.49
46	0.49202	0.45700	542.64
47	0.49217	0.46709	542.80
48	0.49232	0.47718	542.95
49	0.49247	0.48727	543.12
50	0.49263	0.49736	543.28
51	0.49278	0.50744	543.44
52	0.49294	0.51753	543.61
53	0.49311	0.52761	543.78
54	0.49328	0.53770	543.96
55	0.49344	0.54778	544.14
56	0.49362	0.55786	544.32
57	0.49380	0.56794	544.52
58	0.49395	0.57602	544.71
59	0.49418	0.58809	544.91
60	0.49437	0.59817	545.11
61	0.49457	0.60824	545.33
62	0.49479	0.61832	545.55
63	0.49500	0.62835	545.77
64	0.49522	0.63846	546.01

65	0.49546	0.64852	546.25
66	0.49570	0.65655	546.51
67	0.49595	0.66655	546.77
68	0.49622	0.67671	547.05
69	0.49649	0.68977	547.34
70	0.49678	0.69883	547.65
71	0.49708	0.7089	547.97
72	0.49741	0.71894	548.30
73	0.49775	0.72695	548.66
74	0.49810	0.73504	549.04
75	0.49848	0.74908	549.43
76	0.49885	0.75512	549.86
77	0.49932	0.76916	550.31
78	0.49979	0.77920	550.80
79	0.50028	0.78623	551.32
80	0.50082	0.79926	551.89
81	0.50141	0.80925	552.50
82	0.50205	0.81530	553.18
83	0.50275	0.82931	553.91
84	0.50342	0.83532	554.72
85	0.50438	0.84532	555.62
86	0.50534	0.85932	556.63
87	0.50642	0.86631	557.77
88	0.50766	0.87928	559.06
89	0.50909	0.88525	560.55
90	0.51076	0.89521	562.30
91	0.51273	0.90915	564.37
92	0.51512	0.91607	566.86
93	0.51807	0.92898	569.94
94	0.52182	0.93885	573.86
95	0.52677	0.94870	579.02
96	0.53364	0.95849	586.16
97	0.54386	0.96222	596.77
98	0.56082	0.97783	614.29
99	0.59495	0.98722	649.27
100	0.70507	0.99592	759.36
101	1.09185	1.00000	1105.00

T <sub>0</sub>	P <sub>0</sub> INIT	Y	U	H
527.24	2334.45	0.0	126.54	
		25.34	126.54	
		50.07	126.54	
		75.37	126.55	
		100.32	126.57	
		124.87	126.58	
		149.59	126.60	
		174.06	126.63	
		199.30	126.66	
		222.17	126.69	
		245.58	126.72	
		268.45	126.76	
		293.69	126.79	
		312.20	126.83	
		332.88	126.87	
		352.65	126.91	
		371.43	126.95	
		399.15	127.00	
		405.74	127.04	
		421.17	127.07	
		435.40	127.11	
		460.25	127.16	
		480.39	127.24	
		495.13	127.28	
		508.01	127.32	
		516.63	127.34	
		526.72	127.38	
		530.97	127.39	
		532.50	127.39	
		533.13	127.40	

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